

# Contents

Chapter 1. Introduction	1
1.1. The Kronecker problem	1
1.2. The basis-theoretic version of the Kronecker problem	2
1.3. Canonical bases connect quantum Schur-Weyl duality with RSK	3
1.4. The nonstandard quantum group and Hecke algebra	5
1.5. Towards an upper canonical basis for $\check{X}^{\otimes r}$	8
1.6. The approach of Adsul, Sohoni, and Subrahmanyam	9
1.7. A global crystal basis for $\check{X}_\nu$	10
1.8. Organization	11
Chapter 2. Basic concepts and notation	13
2.1. General notation	13
2.2. Tensor products	14
2.3. Words and tableaux	14
2.4. Cells	15
2.5. Comodules	15
2.6. Dually paired Hopf algebras	17
Chapter 3. Hecke algebras and canonical bases	19
3.1. The upper canonical basis of $\mathcal{H}(W)$	19
3.2. Cells in type $A$	20
Chapter 4. The quantum group $GL_q(V)$	21
4.1. The quantized enveloping algebra $U_q(\mathfrak{g}_V)$	21
4.2. FRT-algebras	21
4.3. The quantum coordinate algebra $\mathcal{O}(M_q(V))$	22
4.4. The quantum determinant and the Hopf algebra $\mathcal{O}(GL_q(V))$	24
4.5. A reduction system for $\mathcal{O}(M_q(V))$	25
4.6. Compactness, unitary transformations	25
4.7. Representations of $GL_q(V)$	26
Chapter 5. Bases for $GL_q(V)$ modules	27
5.1. Gelfand-Tsetlin bases and Clebsch-Gordon coefficients	27
5.2. Crystal bases	28
5.3. Global crystal bases	28
5.4. Projected based modules	30
5.5. Tensor products of based modules	31
Chapter 6. Quantum Schur-Weyl duality and canonical bases	33

6.1. Commuting actions on $\mathbf{T} = V^{\otimes r}$	33
6.2. Upper canonical basis of $\mathbf{T}$	34
6.3. Graphical calculus for $U_q(\mathfrak{gl}_2)$ -modules	35
Chapter 7. Notation for $GL_q(V) \times GL_q(W)$	39
Chapter 8. The nonstandard coordinate algebra $\mathcal{O}(M_q(\check{X}))$	41
8.1. Definition of $\mathcal{O}(M_q(\check{X}))$	41
8.2. Nonstandard symmetric and exterior algebras	43
8.3. Explicit product formulae	47
8.4. Examples and computations for $\mathcal{O}(M_q(\check{X}))$	48
Chapter 9. Nonstandard determinant and minors	51
9.1. Definitions	51
9.2. Nonstandard minors in the two-row case	52
9.3. Symmetry of the determinants and minors	54
9.4. Formulae for nonstandard minors	56
Chapter 10. The nonstandard quantum groups $GL_q(\check{X})$ and $U_q(\check{X})$	59
10.1. Hopf structure	59
10.2. Compact real form	60
10.3. Complete reducibility	61
Chapter 11. The nonstandard Hecke algebra $\check{\mathcal{H}}_r$	63
11.1. Definition of $\check{\mathcal{H}}_r$ and basic properties	63
11.2. Semisimplicity of $K\check{\mathcal{H}}_r$	65
11.3. Representation theory of $S^2\check{\mathcal{H}}_r$	66
11.4. Some representation theory of $\check{\mathcal{H}}_r$	66
11.5. The sign representation in the canonical basis	68
11.6. The algebra $\check{\mathcal{H}}_3$	69
11.7. A canonical basis of $\check{\mathcal{H}}_3$	70
11.8. The algebra $\check{\mathcal{H}}_4$	72
Chapter 12. Nonstandard Schur-Weyl duality	73
12.1. Nonstandard Schur-Weyl duality	73
12.2. Consequences for the corepresentation theory of $\mathcal{O}(M_q(\check{X}))$	74
12.3. The two-row, $r = 3$ case	75
Chapter 13. Nonstandard representation theory in the two-row case	77
13.1. The Hopf algebra $U_q^\tau$	77
13.2. The Hopf algebra $\mathcal{O}_q^\tau$	78
13.3. Representation theory of $U_q^\tau$ and $\mathcal{O}_q^\tau$	78
13.4. Schur-Weyl duality between $U_q^\tau$ and $S^2\check{\mathcal{H}}_r$	79
13.5. Upper based $U_q^\tau$ -modules	79
13.6. The nonstandard two-row case	80
Chapter 14. A canonical basis for $\check{Y}_\alpha$	83
14.1. Nonstandard columns label a canonical basis for $\check{\Lambda}^r \check{X}$	83
14.2. Nonstandard tabloids label a canonical basis of $\check{Y}_\alpha$	86
14.3. The action of the Kashiwara operators and $\tau$ on NST	89

Chapter 15. A global crystal basis for two-row Kronecker coefficients	91
15.1. Invariants	91
15.2. Two-column moves	93
15.3. Invariant moves	98
15.4. Nonstandard tabloid classes	98
15.5. Justifying the combinatorics	102
15.6. Explicit formulae for nonintegral $\text{NST}(\beta, \triangleright\gamma, \delta)$	105
15.7. A basis for the two-row Kronecker problem	107
Chapter 16. Straightened NST and semistandard tableaux	115
16.1. Lexicographic order on NST	115
16.2. Bijection with semistandard tableaux	118
16.3. Invariant-free straightened highest weight NST	121
Chapter 17. A Kronecker graphical calculus and applications	123
17.1. Kronecker graphical calculus	123
17.2. Action of the Chevalley generators on $+\text{HNSTC}$	126
17.3. The action of $\tau$ on $+\text{HNSTC}$	133
Chapter 18. Explicit formulae for Kronecker coefficients	135
18.1. Invariant-free Kronecker coefficients and explicit formulae	135
18.2. The symmetric and exterior Kronecker coefficients	137
18.3. Comparisons with other formulae	139
Chapter 19. Future work	141
19.1. A canonical basis for $\check{X}^{\otimes r}$	141
19.2. Defining $\check{X}_\nu$ outside the two-row case	147
Acknowledgments	148
Appendix A. Reduction system for $\mathcal{O}(M_q(\check{X}))$	149
Appendix B. The Hopf algebra $\mathcal{O}_q^\tau$	153
Bibliography	157