

Contents

Preface	xv
Author biographies	xvii
1 Introduction	1
1.1 Introduction to financial derivatives	2
1.2 Financial derivatives — what's the big deal?	6
1.3 Stylized facts	9
1.3.1 No autocorrelation in returns	10
1.3.2 Unconditional heavy tails	10
1.3.3 Gain/loss asymmetry	10
1.3.4 Aggregational Gaussianity	11
1.3.5 Volatility clustering	12
1.3.6 Conditional heavy tails	12
1.3.7 Significant autocorrelation for absolute returns	12
1.3.8 Leverage effects	13
1.4 Overview	14
2 Fundamentals	17
2.1 Interest rates	18
2.1.1 Future and present value of a single payment	18
2.1.2 Annuities	19
2.1.3 Future value of an annuity	19
2.1.4 Present value of a unit annuity	20
2.2 Cash flows	21
2.3 Continuously compounded interest rates	25
2.4 Interest rate options: caps and floors	27
2.5 Notes	32
2.6 Problems	32
3 Discrete time finance	37
3.1 The binomial one-period model	37
3.2 One-period model	39
3.2.1 Risk-neutral probabilities	40
3.2.2 Complete and incomplete markets	41

3.3	Multiperiod model	44
3.3.1	σ -algebras and information sets	47
3.3.2	Financial multiperiod markets	49
3.3.3	Martingale measures	50
3.4	Notes	53
3.5	Problems	53
4	Linear time series models	57
4.1	Introduction	57
4.2	Linear systems in the time domain	59
4.3	Linear stochastic processes	62
4.4	Linear processes with a rational transfer function	63
4.4.1	ARMA process	63
4.4.2	ARIMA process	64
4.4.3	Seasonal models	65
4.5	Autocovariance functions	66
4.5.1	Autocovariance function for ARMA processes	66
4.6	Prediction in linear processes	67
4.7	Problems	69
5	Nonlinear time series models	73
5.1	Introduction	73
5.2	Aim of model building	73
5.3	Qualitative properties of the models	74
5.3.1	Volterra series expansion	74
5.3.2	Generalized transfer functions	75
5.4	Parameter estimation	77
5.4.1	Maximum likelihood estimation	77
5.4.1.1	Cramér–Rao bound	78
5.4.1.2	The likelihood ratio test	79
5.4.2	Quasi-maximum likelihood	79
5.4.3	Generalized method of moments	80
5.4.3.1	GMM and moment restrictions	80
5.4.3.2	Standard error of the estimates	81
5.4.3.3	Estimation of the weight matrix	81
5.4.3.4	Nested tests for model reduction	82
5.5	Parametric models	82
5.5.1	Threshold and regime models	84
5.5.1.1	Self-exciting threshold AR (SETAR)	84
5.5.1.2	Self-exciting threshold ARMA (SETARMA)	86
5.5.1.3	Open loop threshold AR (TARSO)	86
5.5.1.4	Smooth threshold AR (STAR)	86

5.5.1.5	Hidden Markov models and related models	87
5.5.2	Models with conditional heteroscedasticity (ARCH)	89
5.5.2.1	ARCH regression model	89
5.5.2.2	GARCH model	90
5.5.2.3	EGARCH model	91
5.5.2.4	FIGARCH model	92
5.5.2.5	ARCH-M model	92
5.5.2.6	SW-ARCH model	93
5.5.2.7	General remarks on ARCH models	93
5.5.2.8	Multivariate GARCH models	95
5.5.3	Stochastic volatility models	96
5.6	Model identification	98
5.7	Prediction in nonlinear models	98
5.8	Applications of nonlinear models	99
5.8.1	Electricity spot prices	99
5.8.2	Comparing ARCH models	100
5.9	Problems	101
6	Kernel estimators in time series analysis	103
6.1	Non-parametric estimation	103
6.2	Kernel estimators for time series	103
6.2.1	Introduction	103
6.2.2	Kernel estimator	104
6.2.3	Central limit theorems	105
6.3	Kernel estimation for regression	106
6.3.1	Estimator for regression	106
6.3.2	Product kernel	107
6.3.3	Non-parametric estimation of the pdf	108
6.3.4	Non-parametric LS	108
6.3.5	Bandwidth	108
6.3.6	Selection of bandwidth — cross validation	109
6.3.7	Variance of the non-parametric estimates	109
6.4	Applications of kernel estimators	110
6.4.1	Non-parametric estimation of the conditional mean and variance	110
6.4.2	Non-parametric estimation of non-stationarity — an example	111
6.4.3	Non-parametric estimation of dependence on external variables — an example	113
6.4.4	Non-parametric GARCH models	114
6.5	Notes	116

7	Stochastic calculus	117
7.1	Dynamical systems	118
7.2	The Wiener process	120
7.3	Stochastic Integrals	122
7.4	Itô stochastic calculus	125
7.5	Extensions to jump processes	130
7.6	Problems	136
8	Stochastic differential equations	139
8.1	Stochastic Differential Equations	140
8.1.1	Existence and uniqueness	142
8.1.2	Itô formula	147
8.1.3	Multivariate SDEs	149
8.1.4	Stratonovitch SDE	151
8.2	Analytical solution methods	152
8.2.1	Linear, univariate SDEs	152
8.3	Feynman–Kac representation	156
8.4	Girsanov measure transformation	159
8.4.1	Measure theory	159
8.4.2	Radon–Nikodym theorem	161
8.4.3	Girsanov transformation	164
8.4.4	Maximum likelihood estimation for continuously observed diffusions	168
8.5	Notes	171
8.6	Problems	172
9	Continuous-time security markets	175
9.1	From discrete to continuous time	175
9.2	Classical arbitrage theory	177
9.2.1	Black–Scholes formula	181
9.2.2	Hedging strategies	183
9.2.2.1	Quadratic hedging	184
9.3	Modern approach using martingale measures	185
9.4	Pricing	189
9.5	Model extensions	190
9.6	Computational methods	191
9.6.1	Fourier methods	192
9.7	Problems	193
10	Stochastic interest rate models	195
10.1	Gaussian one-factor models	196
10.1.1	Merton model	196
10.1.2	Vasicek model	197
10.2	A general class of one-factor models	198

10.3	Time-dependent models	201
10.3.1	Ho–Lee	201
10.3.2	Black–Derman–Toy	201
10.3.3	Hull–White	201
	10.3.3.1 CIR++ model	202
10.4	Multifactor and stochastic volatility models	202
10.4.1	Stochastic volatility models	204
10.4.2	Affine Term Structure models	205
10.5	Notes	206
10.6	Problems	206
11	Term structure of interest rates	209
11.1	Basic concepts	210
11.1.1	Known interest rates	210
11.1.2	Discrete dividends	212
11.1.3	Yield curve	214
11.1.4	Stochastic interest rates	217
11.2	Classical approach	221
11.2.1	Exogenous specification of the market price of risk	227
11.2.2	Illustrative example	228
11.2.3	Modern approach	231
11.3	Term structure for specific models	232
11.3.1	Example 1: The Vasicek model	235
11.3.2	Example 2: The Ho–Lee model	237
11.3.3	Example 3: The Cox–Ingersoll–Ross model	238
11.3.4	Multifactor models	239
11.4	Heath–Jarrow–Morton framework	240
11.5	Credit models	245
11.5.1	Intensity models	245
11.6	Estimation of the term structure — curve-fitting	246
11.6.1	Polynomial methods	247
11.6.2	Decay functions	247
11.6.3	Nelson–Siegel method	247
11.7	Notes	249
11.8	Problems	250
12	Discrete time approximations	253
12.1	Stochastic Taylor expansion	253
12.2	Convergence	254
12.3	Discretization schemes	255
12.3.1	Strong Taylor approximations	255
12.3.1.1	Explicit Euler scheme	255
12.3.1.2	Milstein scheme	256
12.3.1.3	The order 1.5 strong Taylor scheme	256

12.3.2	Weak Taylor approximations	257
12.3.2.1	The order 2.0 weak Taylor scheme	257
12.3.3	Exponential approximation	257
12.4	Multilevel Monte Carlo	258
12.5	Simulation of SDEs	259
13	Parameter estimation in discretely observed SDEs	265
13.1	Introduction	265
13.2	High frequency methods	266
13.3	Approximate methods for linear and non-linear models	269
13.4	State dependent diffusion term	269
13.4.1	A transformation approach	269
13.5	MLE for non-linear diffusions	271
13.5.1	Simulation-based estimators	271
13.5.1.1	Jump diffusions	272
13.5.2	Numerical methods for the Fokker–Planck equation	273
13.5.3	Series expansion	273
13.6	Generalized method of moments	274
13.6.1	GMM and moment restrictions	275
13.7	Model validation for discretely observed SDEs	277
13.7.1	Generalized Gaussian residuals	277
13.7.1.1	Case study	278
13.8	Problems	280
14	Inference in partially observed processes	283
14.1	Introduction	283
14.2	Model	284
14.3	Exact filtering	285
14.3.1	Prediction	285
14.3.1.1	Scalar case	285
14.3.1.2	General case	286
14.3.2	Updating	287
14.4	Conditional moment estimators	288
14.4.1	Prediction and updating	288
14.5	Kalman filter	289
14.6	Approximate filters	290
14.6.1	Truncated second order filter	290
14.6.2	Linearized Kalman filter	291
14.6.3	Extended Kalman filter	291
14.6.4	Statistically linearized filter	293
14.6.5	Non-linear models	294
14.6.6	Linear time-varying models	295
14.6.7	Linear time-invariant models	295
14.6.8	Case: Affine term structure models	296

14.7	State filtering and prediction	296
14.7.1	Linear models	297
	14.7.1.1 Linear time-varying models	297
	14.7.1.2 Linear time-invariant models	297
14.7.2	The system equation in discrete time	298
14.7.3	Non-linear models	299
14.8	Unscented Kalman Filter	300
14.9	A maximum likelihood method	302
14.10	Sequential Monte Carlo filters	305
14.10.1	Optimal filtering	306
14.10.2	Bootstrap filter	307
14.10.3	Parameter estimation	309
14.11	Application of non-linear filters	310
14.11.1	Sequential calibration of options	310
14.11.2	Computing Value at Risk in a stochastic volatility model	314
14.11.3	Extended Kalman filtering applied to bonds	315
14.11.4	Case 1: A Wiener process	318
14.11.5	Case 2: The Vasicek model	319
14.12	Problems	321
A	Projections in Hilbert spaces	323
A.1	Introduction	323
A.2	Hilbert spaces	324
A.3	The projection theorem	325
	A.3.1 Prediction equations	328
A.4	Conditional expectation and linear projections	329
A.5	Kalman filter	332
A.6	Projections in \mathbb{R}^n	334
B	Probability theory	337
B.1	Measures and σ -algebras	337
B.2	Partitions and information	338
B.3	Conditional expectation	339
B.4	Notes	343
	Bibliography	345
	Index	361