
Contents

Preface page ix

Part I	Theory	1
1	Basic probability concepts	3
1.1	Random experiments and probabilities	3
1.2	Conditional probabilities and independence	9
1.3	Random variables	14
1.4	Conditional expectations	28
1.5	Problems	34
2	Stochastic processes	38
2.1	Definitions and general results	38
2.2	Stopping times	46
2.3	Discrete time martingales	50
2.4	Doob decomposition	56
2.5	Continuous time martingales	59
2.6	Doob–Meyer decomposition	62
2.7	Brownian motion	70
2.8	Brownian motion process with drift	72
2.9	Brownian paths	72
2.10	Poisson process	75
2.11	Problems	75
3	Stochastic calculus	79
3.1	Introduction	79
3.2	Quadratic variations	80
3.3	Simple examples of stochastic integrals	87
3.4	Stochastic integration with respect to a Brownian motion	90
3.5	Stochastic integration with respect to general martingales	94
3.6	The Itô formula for semimartingales	97
3.7	The Itô formula for Brownian motion	108
3.8	Representation results	115
3.9	Random measures	123
3.10	Problems	127

4	Change of measures	131
4.1	Introduction	131
4.2	Measure change for discrete time processes	134
4.3	Girsanov's theorem	145
4.4	The single jump process	150
4.5	Change of parameter in poisson processes	157
4.6	Poisson process with drift	161
4.7	Continuous-time Markov chains	163
4.8	Problems	165
Part II	Applications	167
5	Kalman filtering	169
5.1	Introduction	169
5.2	Discrete-time scalar dynamics	169
5.3	Recursive estimation	169
5.4	Vector dynamics	175
5.5	The EM algorithm	177
5.6	Discrete-time model parameter estimation	178
5.7	Finite-dimensional filters	180
5.8	Continuous-time vector dynamics	190
5.9	Continuous-time model parameters estimation	196
5.10	Direct parameter estimation	206
5.11	Continuous-time nonlinear filtering	211
5.12	Problems	215
6	Financial applications	217
6.1	Volatility estimation	217
6.2	Parameter estimation	221
6.3	Filtering a price process	222
6.4	Parameter estimation for a modified Kalman filter	223
6.5	Estimating the implicit interest rate of a risky asset	229
7	A genetics model	235
7.1	Introduction	235
7.2	Recursive estimates	235
7.3	Approximate formulae	239
8	Hidden populations	242
8.1	Introduction	242
8.2	Distribution estimation	243
8.3	Parameter estimation	246
8.4	Pathwise estimation	247
8.5	A Markov chain model	248

8.6	Recursive parameter estimation	250
8.7	A tags loss model	250
8.8	Gaussian noise approximation	253
	References	255
	Index	257