

---

# Contents

<b>1</b>	<b>Nonparametric estimators</b>	<b>1</b>
1.1	Examples of nonparametric models and problems	1
1.2	Kernel density estimators	2
1.2.1	Mean squared error of kernel estimators	4
1.2.2	Construction of a kernel of order $\ell$	10
1.2.3	Integrated squared risk of kernel estimators	12
1.2.4	Lack of asymptotic optimality for fixed density	16
1.3	Fourier analysis of kernel density estimators	19
1.4	Unbiased risk estimation. Cross-validation density estimators	27
1.5	Nonparametric regression. The Nadaraya–Watson estimator	31
1.6	Local polynomial estimators	34
1.6.1	Pointwise and integrated risk of local polynomial estimators	37
1.6.2	Convergence in the sup-norm	42
1.7	Projection estimators	46
1.7.1	Sobolev classes and ellipsoids	49
1.7.2	Integrated squared risk of projection estimators	51
1.7.3	Generalizations	57
1.8	Oracles	59
1.9	Unbiased risk estimation for regression	61
1.10	Three Gaussian models	65
1.11	Notes	69
1.12	Exercises	72
<b>2</b>	<b>Lower bounds on the minimax risk</b>	<b>77</b>
2.1	Introduction	77
2.2	A general reduction scheme	79
2.3	Lower bounds based on two hypotheses	81
2.4	Distances between probability measures	83
2.4.1	Inequalities for distances	86
2.4.2	Bounds based on distances	90

2.5	Lower bounds on the risk of regression estimators at a point . . . . .	91
2.6	Lower bounds based on many hypotheses . . . . .	95
2.6.1	Lower bounds in $L_2$ . . . . .	102
2.6.2	Lower bounds in the sup-norm . . . . .	108
2.7	Other tools for minimax lower bounds . . . . .	110
2.7.1	Fano's lemma . . . . .	110
2.7.2	Assouad's lemma . . . . .	116
2.7.3	The van Trees inequality . . . . .	120
2.7.4	The method of two fuzzy hypotheses . . . . .	125
2.7.5	Lower bounds for estimators of a quadratic functional . . . . .	128
2.8	Notes . . . . .	131
2.9	Exercises . . . . .	133
<b>3</b>	<b>Asymptotic efficiency and adaptation . . . . .</b>	<b>137</b>
3.1	Pinsker's theorem . . . . .	137
3.2	Linear minimax lemma . . . . .	140
3.3	Proof of Pinsker's theorem . . . . .	146
3.3.1	Upper bound on the risk . . . . .	146
3.3.2	Lower bound on the minimax risk . . . . .	147
3.4	Stein's phenomenon . . . . .	155
3.4.1	Stein's shrinkage and the James–Stein estimator . . . . .	157
3.4.2	Other shrinkage estimators . . . . .	162
3.4.3	Superefficiency . . . . .	165
3.5	Unbiased estimation of the risk . . . . .	166
3.6	Oracle inequalities . . . . .	174
3.7	Minimax adaptivity . . . . .	179
3.8	Inadmissibility of the Pinsker estimator . . . . .	180
3.9	Notes . . . . .	185
3.10	Exercises . . . . .	187
<b>Appendix</b>	. . . . .	<b>191</b>
<b>Bibliography</b>	. . . . .	<b>203</b>
<b>Index</b>	. . . . .	<b>211</b>