Contents

Preface to the English Translation xi

Prefaces to the German Editions xiii
Preface to the first German Edition xiii
Preface to the second German Edition xiv
Preface to the third German Edition xiv

Part 1. Prologue 1

Chapter 1. Math Becomes a Cult—Description of a Hope
GERO VON RANDOW 3

Part 2. Case Studies 5

Chapter 2. The Mathematics of the Compact Disc
JACK H. VAN LINT 7
Words and codes 7
A simple example 7
From music to audiobits 9
Reed–Solomon codes 9
The compact disc 12
References 14

Chapter 3. Image Processing and Imaging for Operation Planning in Liver Surgery
HEINZ-OTTO PEITGEN, CARL EVERTSZ, BERNHARD PREIM, DIRK SELLE, THOMAS SCHINDEWOLF, AND WOLF SPINDLER 15
1. Introduction 15
2. Medical background 16
3. Architecture of a surgery planning system 17
4. Liver and tumor segmentation 18
5. Vessel segmentation and analysis 20
6. Visualization and exploration of the analysed data 23
7. Summary 25
8. Prospect 26
References 26

Chapter 4. The Quickest Path to the Goal
RALF BORNDÖRFER, MARTIN GRÖTSCHEL AND ANDREAS LÖBEL 27
1. Historical overture 27

2. Combinatorics of shortest paths 33
3. Combinations of paths 43
4. Outlook 49
5. Further reading 50
6. Solutions to the questions 51

Chapter 5. Romeo and Juliet, Spontaneous Pattern Formation, and Turing's Instability
BERNOLD FIEDLER 53
1. Turing dreams 53
2. Romeo and Juliet 54
3. Roberto and Julietta 55
4. When sisters gossip ... 57
5. ... and brothers brag 60
6. Turing's theorem 63
7. Mathematical summary 64
8. Outlook 66
References 69

Chapter 6. Mathematics and Intelligent Materials
STEFAN MÜLLER 71
Mathematics as a key technology 71
Metals with memory 71
Memory and microstructure 71
Microstructures everywhere 74
Microstructures as optimal forms 75
Mathematical chance helps—Young measures 76
Design of new materials through mathematics 77
Future challenges: multiscale mathematics, or the bridging from atoms to materials 77
Protein folding, rough energy landscapes, and optimization 80
References 80

Chapter 7. Discrete Tomography: From Battleship to Nanotechnology
PETER GRITZMANN 81
A glimpse into the human body 81
Behind the teacher's back 82
Duty rosters and data security 87
Reconstruction of crystalline structures 87
Uniqueness theorems 91
Complexity and algorithms 95
Stability 96

Chapter 8. Reflections on Reflections
JÜRGEN RICHTER-GEBERT 99
Childhood memories 99
1. Good angles, bad angles 99
2. One, two, three ... infinity 101
3. Kaleidoscopes—beauty-viewers 101
4. Number games 103

5. Light billiards, anti-stealth-boats and egoist mirrors 104
6. The perfect display cupboard 106
7. The way from the right angle 108
8. Platonic beauties 110
9. Christmas chaos 111
10. Circle inversions 112
11. A new universe 113
12. To infinity, and beyond 116
13. Reading and surfing tips 118

Part 3. Current Topics 121

Chapter 9. The Role of Mathematics in the Financial Markets
WALTER SCHACHERMAYER 123
References 133

Chapter 10. Electronic Money:
An Impossibility or Already a Reality?
ALBRECHT BEUTELSPACHER 135
1. Introduction 135
2. What is money? 135
3. Cryptographic mechanisms 136
4. Electronic money: the basic scheme 138
5. Double spending 139
6. Extra properties 140
Summary 141
References 141

Chapter 11. Spheres in the Computer—the Kepler Conjecture
MARTIN HENK AND GÜNTER M. ZIEGLER 143
A really hard nut 143
In the plane 145
Into the third dimension 152
A scandalous situation 156
A recipe? 157
Computer versus Kepler 161
Problems, problems 161
References 163

Chapter 12. How Do Quanta Compute?
The New World of the Quantum Computer
EIHRHARD BEHRENDS 165
1. Why are prime numbers important in cryptography? 166
2. A mathematical preparation: period lengths 167
3. Some quantum mechanics 168
4. Qbits: the components of a quantum computer 170
5. How does one factorize large numbers with a quantum computer? 171
6. Summary 173

Chapter 13. Fermat's Last Theorem—the Solution of a 300 Year Old Problem
JÜRG KRAMER 175
1. Introduction 175
2. How did Fermat come to his Conjecture? 175
3. The period between 1637 and 1980 177
4. The three worlds 178
5. The bridges between the three worlds 181
6. The anti-Fermat world does not exist 182
References 183

Chapter 14. A Short History of the Nash Equilibrium
KARL SIGMUND 185
Does Sherlock Holmes have a chance? 185
The art of the bluff 186
Maximin solutions 188
The Nash equilibrium 189
Ideas from evolution theory 190
The prisoners' dilemma 191
Tit for Tat 192
Altruism versus self-interest 193

Chapter 15. Mathematics in the Climate of Global Change
RUPERT KLEIN 197
Why climate and climate impact research? 197
Complexities 199
"Story exercises" 202
Multiple scales 204
Approximate solutions and missing lattice points 206
Multiscale asymptotics for the oscillator with small mass and damping 208
Hurricanes: an example in multiscale phenomena 212
Conclusion 214
References 215

Part 4. The Central Theme 217

Chapter 16. Prime Numbers, Secret Codes and the Boundaries of Computability
MARTIN AIGNER 219
1. Prime numbers 219
2. Secret codes 221
3. Boundaries of computability 224
References 226

Chapter 17. The Mathematics of Knots
ELMAR VOGT 227
History 227
Wild and tame knots and the search for the right mathematical concept 231

Polygonal knots
The Reidemeister approach to knot theory 235
There are true knots 238
Some families of knots 244

Chapter 18. On Soap Bubbles
DIRK FERUS 251
References and picture credits 259

Chapter 19. Heat Diffusion, the Structure of Space, and the Poincaré Conjecture
KLAUS ECKER 261
1. Introduction 261
2. Geometry and topology of surfaces 263
3. Geometry and topology of three-dimensional spaces 276
4. Heat diffusion and the geometry of curves 285
5. Ricci flow, geometrization and the Poincaré Conjecture 288
6. Conclusion 296
References 296

Chapter 20. Chance and Mathematics: a Late Love
EHRHARD BEHRENDS 299
1. How did it start? 299
2. How is it done today? 300
3. Fundamental concepts 302
4. Games of chance 305
5. Randomness vanishes at infinity 307
6. The productive role of chance 309
7. Chance in the microcosmos 310
8. Philosophical 312

Part 5. Epilogue 315

Chapter 21. The Prospects for Mathematics in a Multi-Media Civilization
PHILIP J. DAVIS 317
Poincaré's predictions 318
What will pull mathematics into the future? 318
The inner texture (or soul) of mathematics 324
A personal illumination 329
References 330