

Contents

Introduction	ix
0.1. Background	ix
0.2. Main results	xii
0.3. The connection with generalised Morse functions and Part II	xiv
0.4. Acknowledgements	xvii
Chapter 1. Definitions and Preliminary Results	1
1.1. Isotopy and concordance in the space of metrics of positive scalar curvature	1
1.2. Warped product metrics on the sphere	2
1.3. Torpedo metrics on the disk	4
1.4. Doubly warped products and mixed torpedo metrics	5
1.5. Inducing a mixed torpedo metric with an embedding	9
Chapter 2. Revisiting the Surgery Theorem	11
2.1. Surgery and cobordism	11
2.2. Surgery and positive scalar curvature	12
2.3. Outline of the proof of Theorem 2.3	14
2.4. Part 1 of the proof: Curvature formulae for the first deformation	15
2.5. Part 2 of the proof: A continuous bending argument	17
2.6. Part 3 of the proof: Isotoping to a standard product	26
2.7. Applying Theorem 2.3 over a compact family of psc-metrics	29
2.8. The proof of Theorem 2.2 (The Improved Surgery Theorem)	31
Chapter 3. Constructing Gromov-Lawson Cobordisms	35
3.1. Morse Theory and admissible Morse functions	35
3.2. A reverse Gromov-Lawson cobordism	40
3.3. Continuous families of Morse functions	41
Chapter 4. Constructing Gromov-Lawson Concordances	45
4.1. Applying the Gromov-Lawson technique over a pair of cancelling surgeries	45
4.2. Cancelling Morse critical points: The Weak and Strong Cancellation Theorems	47
4.3. A strengthening of Theorem 4.2	48
4.4. Standardising the embedding of the second surgery sphere	49
Chapter 5. Gromov-Lawson Concordance Implies Isotopy for Cancelling Surgeries	51
5.1. Connected sums of psc-metrics	51
5.2. An analysis of the metric g'' , obtained from the second surgery	51

5.3. The proof of Theorem 5.1	53
Chapter 6. Gromov-Lawson Concordance Implies Isotopy in the General Case	65
6.1. A weaker version of Theorem 0.8	65
6.2. The proof of the main theorem	67
Appendix: Curvature Calculations from the Surgery Theorem	71
Bibliography	79