Contents

Preface ix

Chapter 1. Measure and Integral 1

§1. Measure 6

§2. The Lebesgue Integral 22

§3. Foundations of Probability Theory 31

§4. Literature 36

Bibliography 37

Chapter 2. High-Dimensional Geometry and Measure Concentration 39

§1. Peculiarities of Large Dimensions 41

§2. The Brunn–Minkowski Inequality and Euclidean Isoperimetry 44

§3. The Standard Normal Distribution and the Gaussian Measure 53

§4. Measure Concentration 61

§5. Literature 81

Bibliography 82

Chapter 3. Fourier Analysis 85

§1. Characters 87

§2. The Fourier Transform 94

§3. Two Unexpected Applications 99
§4. Convolution 106
§5. Poisson Summation Formula 109
§6. Influence of Variables 113
§7. Infinite Groups 124
§8. Literature 137
Bibliography 138

Chapter 4. Representations of Finite Groups 141
§1. Basic Definitions and Examples 142
§2. Decompositions into Irreducible Representations 145
§3. Irreducible Decompositions, Characters, Orthogonality 150
§4. Irreducible Representations of the Symmetric Group 160
§5. An Application in Communication Complexity 163
§6. More Applications and Literature 168
Bibliography 169

Chapter 5. Polynomials 173
§1. Rings, Fields, and Polynomials 173
§2. The Schwartz–Zippel Theorem 175
§3. Polynomial Identity Testing 176
§4. Interpolation, Joints, and Contagious Vanishing 180
§5. Varieties, Ideals, and the Hilbert Basis Theorem 185
§6. The Nullstellensatz 188
§7. Bézout's Inequality in the Plane 195
§8. More Properties of Varieties 200
§9. Bézout's Inequality in Higher Dimensions 219
§10. Bounding the Number of Connected Components 226
§11. Literature 232
Bibliography 232

Chapter 6. Topology 235
§1. Topological Spaces and Continuous Maps 236
§2. Bits of General Topology 240
§3. Compactness 247
§4. Homotopy and Homotopy Equivalence 253
§5. The Borsuk–Ulam Theorem 257
§6. Operations on Topological Spaces 262
§7. Simplicial Complexes and Relatives 271
§8. Non-embeddability 283
§9. Homotopy Groups 289
§10. Homology of Simplicial Complexes 301
§11. Simplicial Approximation 309
§12. Homology Does Not Depend on Triangulation 314
§13. A Quick Harvest and Two More Theorems 318
§14. Manifolds 321
§15. Literature 330
Bibliography 330

Index 333