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L^2 Approaches in Several Complex Variables

Development of Oka–Cartan Theory
by L^2 Estimates for the $\bar{\partial}$ Operator

 Springer

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