
Contents

Preface	ix
Introduction	1
1 Topological Invariants and Differential Geometry	6
1.1 <i>Topological Invariants</i>	7
1.2 <i>Fundamental Groups and Covering Spaces</i>	10
1.3 <i>Complex Manifolds and Metrics</i>	13
1.4 <i>Divisors, Line Bundles, the First Chern Class</i>	16
2 Riemann Surfaces, Coverings, and Hypergeometric Functions	23
2.1 <i>Genus and Euler Number</i>	23
2.2 <i>Möbius Transformations</i>	25
2.3 <i>Metric and Curvature</i>	29
2.4 <i>Behavior of the Euler Number under Finite Covering</i>	33
2.5 <i>Finite Subgroups of $\mathrm{PSL}(2, \mathbb{C})$</i>	34
2.6 <i>Gauss Hypergeometric Functions</i>	36
2.7 <i>Triangle Groups</i>	41
2.8 <i>The Hypergeometric Monodromy Group</i>	45
3 Complex Surfaces and Coverings	47
3.1 <i>Coverings Branched over Subvarieties with Transverse Intersections</i>	47
3.2 <i>Divisor Class Group and Canonical Class</i>	49
3.3 <i>Proportionality</i>	54
3.4 <i>Signature</i>	59
3.5 <i>Blowing Up Points</i>	61
4 Algebraic Surfaces and the Miyaoka-Yau Inequality	65
4.1 <i>Rough Classification of Algebraic Surfaces</i>	65
4.2 <i>The Miyaoka-Yau Inequality, I</i>	70
4.3 <i>The Miyaoka-Yau Inequality, II</i>	73
5 Line Arrangements in $\mathbb{P}_2(\mathbb{C})$ and Their Finite Covers	85
5.1 <i>Blowing Up Line Arrangements</i>	87
5.2 <i>Höfer's Formula</i>	88

5.3 Arrangements Annihilating R and Having Equal Ramification along All Lines	92
5.4 Blow-Up of a Singular Intersection Point	99
5.5 Possibilities for the Assigned Weights	103
5.6 Blowing Down Rational Curves and Removing Elliptic Curves	115
5.7 Tables of the Weights Giving $\text{Prop} = 0$	122
6 Existence of Ball Quotients Covering Line Arrangements	126
6.1 Existence of Finite Covers by Ball Quotients of Weighted Configurations: The General Case	128
6.2 Remarks on Orbifolds and b -Spaces	133
6.3 $K_{X''} + D''$ for Weighted Line Arrangements	135
6.4 Existence Question	139
6.5 Ampleness of $K_{X''} + D''$	140
6.6 Log-Terminal Singularities and LCS	145
6.7 Existence Theorem for Line Arrangements	148
6.8 Isotropy Subgroups of the Covering Group	164
7 Appell Hypergeometric Functions	167
7.1 The Action of S_5 on the Blown-Up Projective Plane	168
7.2 Appell Hypergeometric Functions	173
7.3 Arithmetic Monodromy Groups	181
7.4 Some Remarks about the Signature	186
A Torsion-Free Subgroups of Finite Index	189
A.1 Fuchsian Groups	190
A.2 Fenchel's Conjecture	191
A.3 Reduction to Triangle Groups	192
A.4 Triangle Groups	193
B Kummer Coverings	197
Bibliography	205
Index	213