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Non-Divergence Equations Structured on Hörmander Vector Fields: Heat Kernels and Harnack Inequalities

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