

Contents

Preface	v
1 Introduction	1
I Fourier Transformation and Pseudodifferential Operators	
2 Fourier Transformation and Tempered Distributions	9
2.1 Definition and Basic Properties	9
2.2 Rapidly Decreasing Functions – $\mathcal{S}(\mathbb{R}^n)$	13
2.3 Inverse Fourier Transformation and Plancherel’s Theorem	15
2.4 Tempered Distributions and Fourier Transformation	20
2.5 Fourier Transformation and Convolution of Tempered Distributions	23
2.6 Convolution on $\mathcal{S}'(\mathbb{R}^n)$ and Fundamental Solutions	25
2.7 Sobolev and Bessel Potential Spaces	27
2.8 Vector-Valued Fourier-Transformation	30
2.9 Final Remarks and Exercises	33
2.9.1 Further Reading	33
2.9.2 Exercises	34
3 Basic Calculus of Pseudodifferential Operators on \mathbb{R}^n	40
3.1 Symbol Classes and Basic Properties	40
3.2 Composition of Pseudodifferential Operators: Motivation	45
3.3 Oscillatory Integrals	46
3.4 Double Symbols	51

3.5	Composition of Pseudodifferential Operators	54
3.6	Application: Elliptic Pseudodifferential Operators and Parametrixes	57
3.7	Boundedness on $C_b^\infty(\mathbb{R}^n)$ and Uniqueness of the Symbol	63
3.8	Adjoint of Pseudodifferential Operators and Operators in (x, y) -Form	65
3.9	Boundedness on $L^2(\mathbb{R}^n)$ and L^2 -Bessel Potential Spaces	68
3.10	Outlook: Coordinate Transformations and PsDOs on Manifolds	74
3.11	Final Remarks and Exercises	77
3.11.1	Further Reading	77
3.11.2	Exercises	78
II	Singular Integral Operators	
4	Translation Invariant Singular Integral Operators	85
4.1	Motivation	85
4.2	Main Result in the Translation Invariant Case	87
4.3	Calderón–Zygmund Decomposition and the Maximal Operator	91
4.4	Proof of the Main Result in the Translation Invariant Case	95
4.5	Examples of Singular Integral Operators	100
4.6	Mikhlin Multiplier Theorem	107
4.7	Outlook: Hardy spaces and BMO	112
4.8	Final Remarks and Exercises	118
4.8.1	Further Reading	118
4.8.2	Exercises	118
5	Non-Translation Invariant Singular Integral Operators	122
5.1	Motivation	122
5.2	Extension to Non-Translation Invariant and Vector-Valued Singular Integral Operators	124
5.3	Hilbert-Space-Valued Mikhlin Multiplier Theorem	129

5.4	Kernel Representation of a Pseudodifferential Operator	133
5.5	Consequences of the Kernel Representation	140
5.6	Final Remarks and Exercises	143
5.6.1	Further Reading	143
5.6.2	Exercises	144
III	Applications to Function Space and Differential Equations	
6	Introduction to Besov and Bessel Potential Spaces	149
6.1	Motivation	149
6.2	A Fourier-Analytic Characterization of Hölder Continuity	150
6.3	Bessel Potential and Besov Spaces – Definitions and Basic Properties	153
6.4	Sobolev Embeddings	160
6.5	Equivalent Norms	162
6.6	Pseudodifferential Operators on Besov Spaces	164
6.7	Final Remarks and Exercises	168
6.7.1	Further Reading	168
6.7.2	Exercises	168
7	Applications to Elliptic and Parabolic Equations	171
7.1	Applications of the Mihlin Multiplier Theorem	171
7.1.1	Resolvent of the Laplace Operator	171
7.1.2	Spectrum of Multiplier Operators with Homogeneous Symbols	174
7.1.3	Spectrum of a Constant Coefficient Differential Operator . . .	177
7.2	Applications of the Hilbert-Space-Valued Mihlin Multiplier Theorem	180
7.2.1	Maximal Regularity of Abstract ODEs in Hilbert Spaces	180
7.2.2	Hilbert-Space Valued Bessel Potential and Sobolev Spaces . .	185
7.3	Applications of Pseudodifferential Operators	186
7.3.1	Elliptic Regularity for Elliptic Pseudodifferential Operators .	186
7.3.2	Resolvents of Parameter-Elliptic Differential Operators	188
7.3.3	Application of Resolvent Estimates to Parabolic Initial Value Problems	193

7.4	Final Remarks and Exercises	194
7.4.1	Further Reading	194
7.4.2	Exercises	195
IV	Appendix	
A	Basic Results from Analysis	199
A.1	Notation and Functions on \mathbb{R}^n	199
A.2	Lebesgue Integral and L^p -Spaces	201
A.3	Linear Operators and Dual Spaces	206
A.4	Bochner Integral and Vector-Valued L^p -Spaces	209
A.5	Fréchet Spaces	212
A.6	Exercises	216
	Bibliography	217
	Index	221