

Ju-Yi Yen • Marc Yor

Local Times and Excursion Theory for Brownian Motion

A Tale of Wiener and Itô Measures



Springer

Contents

1	Prerequisites	1
1.1	Brownian Motion.....	1
1.2	Some Extensions.....	2
1.3	BM as a Continuous Martingale.....	2
1.4	Girsanov’s Theorem.....	4
1.5	Brownian Bridge.....	5
1.6	The BES(3) Process as a Doob h -Transform of BM.....	7
1.7	The Beta–Gamma Algebra.....	7
1.8	The Law of the Maximum of a Positive Continuous Local Martingale, Which Converges to 0.....	8
1.9	A First Taste of Enlargement Formulae.....	9
1.10	Kolmogorov’s Continuity Criterion.....	9
	References.....	10

Part I Local Times of Continuous Semimartingales

2	The Existence and Regularity of Semimartingale Local Times	13
2.1	From Itô’s Formula to the Occupation Time Formula.....	13
2.2	Regularity of Occupation Times.....	14
2.3	Occupation Times Are Local Times.....	18
2.4	Local Times and the Balayage Formula.....	19
2.5	Some Simple Martingales.....	22
2.6	The Existence of Principal Values Related to Brownian Local Times.....	23
2.7	Some Extensions of Itô’s Formula.....	24
	References.....	27
3	Lévy’s Representation of Reflecting BM and Pitman’s Representation of BES(3)	29
3.1	Lévy’s Identity in Law: The Local Time as a Supremum Process.....	29
3.2	A Solution to Skorokhod’s Embedding Problem.....	30

3.3	Pitman's Representation of BES(3)	33
3.4	A Relation Between (The Above Solution to) Skorokhod's Problem and the Balayage Formula	35
3.5	An Extension of Pitman's Theorem to Brownian Motion with Drift	36
3.6	Skorokhod's Lemma and the Balayage Formula	38
3.7	Seshadri's Remark on the Joint Law of (S_t, B_t)	40
3.8	A Combination of Skorokhod's Lemma and Time-Substitution ...	41
	References	41
4	Paul Lévy's Arcsine Laws	43
4.1	Two Brownian Functionals with the Arcsine Distribution	43
4.2	Two Independent Reflected Brownian Motions.....	44
4.3	Random Brownian Scaling and Absolute Continuity Properties...	45
4.4	The Second Arcsine Law	48
4.5	The Time Spent in \mathbb{R}_+ by a Brownian Bridge	50
4.6	The Law of A_T^+ for More Random Times T and Other Processes than BM	51
	References	53
 Part II Excursion Theory for Brownian Paths		
5	Brownian Excursion Theory: A First Approach	57
5.1	Some Motivations	57
5.2	Itô's Theorem on Excursions	59
5.3	Two Master Formulae (A) and (M).....	60
5.4	Relationship Between Certain Lévy Measures and Itô Measure \mathbf{n}	61
5.5	Two Applications of (A) and (M)	62
	References	64
6	Two Descriptions of \mathbf{n}: Itô's and Williams'	65
6.1	Statements	65
6.2	An Agreement Formula	67
6.3	\mathbf{n} is a Markovian Measure	68
6.4	Proof of Itô's Disintegration (b) in Sect. 6.1	68
6.5	Proof of the Formula (6.4.4) for $\Pi'(\Gamma)$	70
6.6	Proof of the Markovianity of \mathbf{n}	72
6.7	The Formula for Entrance Laws	74
6.8	A (Partial) Proof of Williams' Representation of \mathbf{n}	75
	References	77
7	A Simple Path Decomposition of Brownian Motion Around Time $t = 1$	79
7.1	Another Representation of the Brownian Bridge	79
7.2	The Normalized Brownian Excursion	80

7.3	The Brownian Meander	81
7.4	The Brownian Co-meander	83
7.5	Two Isolation Formulae	85
7.6	Azéma's Martingale and the Brownian Meander	88
	References	91
8	The Laws of, and Conditioning with Respect to,	
	Last Passage Times	93
8.1	The Bessel Case	93
8.2	General Transient Diffusions	93
8.3	Absolute Continuity Relationships up to γ_y	96
8.4	Applications	97
	8.4.1 BM with drift considered up to last passage time	97
	8.4.2 BES process up to last passage time	98
	8.4.3 First hit of 0 by Ornstein–Uhlenbeck process	99
	References	100
9	Integral Representations Relating W and n	101
9.1	Statement of the Main Theorem	101
9.2	Proof of the Theorem	102
9.3	Proof of (9.2.1)	103
	References	104
Part III Some Applications of Excursion Theory		
10	The Feynman–Kac Formula and Excursion Theory	107
10.1	Statement of the FK Formula	107
10.2	Proof of FK via Excursion Theory	108
	References	110
11	Some Identities in Law	111
11.1	On Linear Combinations of Reflected BM and Its Local Time	111
11.2	On the Joint Laws of (S_b, I_b, L_b) and (S_1, I_1, L_1)	114
11.3	Knight's Identity in Law	118
11.4	The Földes–Révész Identity	120
11.5	Cauchy Principal Value of Brownian Local Times	122
11.6	The Agreement Formula and the Functional Equation of the Riemann ζ Function	123
11.7	On Ranked Lengths of Excursions	126
	References	130
	General References	133
	Index	135