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Ordinary Differential Equations with Applications

Second Edition

With 73 Illustrations

 Springer

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