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Visual Quantum Mechanics

Selected Topics with Computer-Generated Animations of Quantum-Mechanical Phenomena

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Contents

Preface v

Chapter 1. Visualization of Wave Functions 1
1.1. Introduction 1
1.2. Visualization of Complex Numbers 2
1.3. Visualization of Complex-Valued Functions 9
1.4. Special Topic: Wave Functions with an Inner Structure 13

Chapter 2. Fourier Analysis 15
2.1. Fourier Series of Complex-Valued Functions 16
2.2. The Hilbert Space of Square-Integrable Functions 21
2.3. The Fourier Transformation 25
2.4. Basic Properties of the Fourier Transform 28
2.5. Linear Operators 29
2.6. Further Results About the Fourier Transformation 34
2.7. Gaussian Functions 38
2.8. Inequalities 41
2.9. Special Topic: Dirac Delta Distribution 45

Chapter 3. Free Particles 49
3.1. The Free Schrödinger Equation 50
3.2. Wave Packets 55
3.3. The Free Time Evolution 59
3.4. The Physical Meaning of a Wave Function 63
3.5. Continuity Equation 70
3.6. Special Topic: Asymptotic Time Evolution 72
3.7. Schrödinger Cat States 75
3.8. Special Topic: Energy Representation 80

Chapter 4. States and Observables 83
4.1. The Hilbert Space of Wave Functions 84
4.2. Observables and Linear Operators 86
4.3. Expectation Value of an Observable 89

4.4. Other Observables 91
4.5. The Commutator of x and p 93
4.6. Electromagnetic Fields 94
4.7. Gauge Fields 97
4.8. Projection Operators 100
4.9. Transition Probability 104

Chapter 5. Boundary Conditions 107
5.1. Impenetrable Barrier 108
5.2. Other Boundary Conditions 110
5.3. Particle in a Box 111
5.4. Eigenvalues and Eigenfunctions 114
5.5. Special Topic: Unit Function in a Dirichlet Box 119
5.6. Particle on a Circle 124
5.7. The Double Slit Experiment 125
5.8. Special Topic: Analysis of the Double Slit Experiment 130

Chapter 6. Linear Operators in Hilbert Spaces 135
6.1. Hamiltonian and Time Evolution 135
6.2. Unitary Operators 138
6.3. Unitary Time Evolution and Unitary Groups 139
6.4. Symmetric Operators 141
6.5. The Adjoint Operator 143
6.6. Self-Adjointness and Stone's Theorem 144
6.7. Translation Group 147
6.8. Weyl Relations 149
6.9. Canonical Commutation Relations 151
6.10. Commutator and Uncertainty Relation 152
6.11. Symmetries and Conservation Laws 153

Chapter 7. Harmonic Oscillator 157
7.1. Basic Definitions and Properties 158
7.2. Eigenfunction Expansion 163
7.3. Solution of the Initial-Value Problem 167
7.4. Time Evolution of Observables 171
7.5. Motion of Gaussian Wave Packets 175
7.6. Harmonic Oscillator in Two and More Dimensions 177
7.7. Theory of the Harmonic Oscillator 179
7.8. Special Topic: More About Coherent States 184
7.9. Special Topic: Mehler Kernel 187

Chapter 8. Special Systems 191
8.1. The Free Fall in a Constant Force Field 192

8.2. Free Fall with Elastic Reflection at the Ground 196
8.3. Magnetic Fields in Two Dimensions 200
8.4. Constant Magnetic Field 202
8.5. Energy Spectrum in a Constant Magnetic Field 205
8.6. Translational Symmetry in a Magnetic Field 207
8.7. Time Evolution in a Constant Magnetic Field 213
8.8. Systems with Rotational Symmetry in Two Dimensions 218
8.9. Spherical Harmonic Oscillator 222
8.10. Angular Momentum Eigenstates in a Magnetic Field 224

Chapter 9. One-Dimensional Scattering Theory 227
9.1. Asymptotic Behavior 227
9.2. Example: Potential Step 231
9.3. Wave Packets and Eigenfunction Expansion 234
9.4. Potential Step: Asymptotic Momentum Distribution 236
9.5. Scattering Matrix 239
9.6. Transition Matrix 241
9.7. The Tunnel Effect 246
9.8. Example: Potential Well 248
9.9. Parity 251

Appendix A. Numerical Solution in One Dimension 257

Appendix B. Movie Index 263
1. Visualization 263
2. Fourier Analysis 264
3. Free Particles 265
4. Boundary Conditions 266
5. Harmonic Oscillator 268
6. Special Systems 270
7. Scattering Theory 272

Appendix C. Other Books on Quantum Mechanics 275

Index 279