

Contents

1 Deformation of Locally Free Actions and Leafwise Cohomology	1
<i>Masayuki Asaoka</i>	
Introduction	1
1.1 Locally free actions and their deformations	3
1.1.1 Locally free actions	3
1.1.2 Rigidity and deformations of actions	5
1.2 Rigidity and deformation of flows	7
1.2.1 Parameter rigidity of locally free \mathbb{R} -actions	7
1.2.2 Deformation of flows	10
1.3 Leafwise cohomology	11
1.3.1 Definition and some basic properties	11
1.3.2 Computation by Fourier analysis	14
1.3.3 Computation by a Mayer–Vietoris argument	16
1.3.4 Other examples	17
1.4 Parameter deformation	19
1.4.1 The canonical 1-form	19
1.4.2 Parameter deformation of \mathbb{R}^p -actions	22
1.4.3 Parameter rigidity of some non-abelian actions	24
1.4.4 A complete deformation for actions of GA	28
1.5 Deformation of orbits	30
1.5.1 Infinitesimal deformation of foliations	30
1.5.2 Hamilton’s criterion for local rigidity	31
1.5.3 Existence of locally transverse deformations	32
1.5.4 Transverse geometric structures	34
Bibliography	37
2 Fundaments of Foliation Theory	41
<i>Aziz El Kacimi Alaoui</i>	
Foreword	41
Part I. Foliations by Example	41
2.1 Generalities	41

2.1.1	Induced foliations	43
2.1.2	Morphisms of foliations	44
2.1.3	Frobenius Theorem	44
2.1.4	Holonomy of a leaf	45
2.2	Transverse structures	46
2.2.1	Lie foliations	47
2.2.2	Transversely parallelizable foliations	48
2.2.3	Riemannian foliations	48
2.2.4	\mathcal{G}/\mathcal{H} -foliations	49
2.2.5	Transversely holomorphic foliations	52
2.3	More examples	53
2.3.1	Simple foliations	53
2.3.2	Linear foliation on the torus \mathbb{T}^2	53
2.3.3	One-dimensional foliations	54
2.3.4	Reeb foliation on the 3-sphere \mathbb{S}^3	54
2.3.5	Lie group actions	55
2.4	Suspension of diffeomorphism groups	58
2.4.1	General construction	58
2.4.2	Examples	59
2.5	Codimension 1 foliations	61
2.5.1	Existence	61
2.5.2	Topological behavior of leaves	62
 Part II. A Digression: Basic Global Analysis		63
2.6	Foliated bundles	64
2.6.1	Examples	65
2.7	Transversely elliptic operators	66
2.8	Examples	70
2.8.1	The basic de Rham complex	70
2.8.2	The basic Dolbeault complex	72
 Part III. Some Open Questions		74
2.9	Transversely elliptic operators	74
2.9.1	Towards a basic index theory	74
2.9.2	Existence of transversely elliptic operators	75
2.9.3	Homotopy invariance of basic cohomology	75
2.10	Complex foliations	75
2.10.1	The $\bar{\partial}_{\mathcal{F}}$ -cohomology	76
2.11	Deformations of Lie foliations	79
2.11.1	Example of a deformation of an Abelian foliation	80
2.11.2	Further questions	80
Bibliography		83

3 Lectures on Foliation Dynamics	87
<i>Steven Hurder</i>	
Introduction	87
3.1 Foliation basics	89
3.2 Topological dynamics	91
3.3 Derivatives	96
3.4 Counting	101
3.5 Exponential complexity	106
3.6 Entropy and exponent	112
3.7 Minimal sets	117
3.8 Classification schemes	120
3.9 Matchbox manifolds	123
3.10 Topological shape	131
3.11 Shape dynamics	134
Appendix A. Homework	136
Bibliography	137
4 Transversal Dirac Operators on Distributions, Foliations, and G-Manifolds	151
<i>Ken Richardson</i>	
Foreword	151
4.1 Introduction to ordinary Dirac operators	152
4.1.1 The Laplacian	152
4.1.2 The ordinary Dirac operator	154
4.1.3 Properties of Dirac operators	157
4.1.4 The Atiyah–Singer Index Theorem	160
4.2 Transversal Dirac operators on distributions	163
4.3 Basic Dirac operators on Riemannian foliations	167
4.3.1 Invariance of the spectrum of basic Dirac operators	167
4.3.2 The basic de Rham operator	170
4.3.3 Poincaré duality and consequences	172
4.4 Natural examples of transversal Dirac operators on G -manifolds	173
4.4.1 Equivariant structure of the orthonormal frame bundle	173
4.4.2 Dirac-type operators on the frame bundle	176
4.5 Transverse index theory for G -manifolds and Riemannian foliations	178
4.5.1 Introduction: the equivariant index	178
4.5.2 Stratifications of G -manifolds	181
4.5.3 Equivariant desingularization	183
4.5.4 The fine decomposition of an equivariant bundle	184
4.5.5 Canonical isotropy G -bundles	186
4.5.6 The equivariant index theorem	187
4.5.7 The basic index theorem for Riemannian foliations	190
Bibliography	195