

---

---

# Contents

|  |            |
|--|------------|
| <b>Preface</b>   | <b>vii</b> |
| <b>1 Introduction and Statement of Main Results</b>            | <b>1</b>   |
| 1.1 Gross–Zagier formula on modular curves . . . . .           | 1          |
| 1.2 Shimura curves and abelian varieties . . . . .             | 2          |
| 1.3 CM points and Gross–Zagier formula . . . . .               | 6          |
| 1.4 Waldspurger formula . . . . .                              | 9          |
| 1.5 Plan of the proof . . . . .                                | 12         |
| 1.6 Notation and terminology . . . . .                         | 20         |
| <b>2 Weil Representation and Waldspurger Formula</b>           | <b>28</b>  |
| 2.1 Weil representation . . . . .                              | 28         |
| 2.2 Shimizu lifting . . . . .                                  | 36         |
| 2.3 Integral representations of the $L$ -function . . . . .    | 40         |
| 2.4 Proof of Waldspurger formula . . . . .                     | 43         |
| 2.5 Incoherent Eisenstein series . . . . .                     | 44         |
| <b>3 Mordell–Weil Groups and Generating Series</b>             | <b>58</b>  |
| 3.1 Basics on Shimura curves . . . . .                         | 58         |
| 3.2 Abelian varieties parametrized by Shimura curves . . . . . | 68         |
| 3.3 Main theorem in terms of projectors . . . . .              | 83         |
| 3.4 The generating series . . . . .                            | 91         |
| 3.5 Geometric kernel . . . . .                                 | 97         |
| 3.6 Analytic kernel and kernel identity . . . . .              | 100        |
| <b>4 Trace of the Generating Series</b>                        | <b>106</b> |
| 4.1 Discrete series at infinite places . . . . .               | 106        |
| 4.2 Modularity of the generating series . . . . .              | 110        |
| 4.3 Degree of the generating series . . . . .                  | 117        |
| 4.4 The trace identity . . . . .                               | 122        |
| 4.5 Pull-back formula: compact case . . . . .                  | 128        |
| 4.6 Pull-back formula: non-compact case . . . . .              | 138        |
| 4.7 Interpretation: non-compact case . . . . .                 | 153        |

|  |            |
|--|------------|
| <b>5 Assumptions on the Schwartz Function</b>                    | <b>171</b> |
| 5.1 Restating the kernel identity . . . . .                      | 171        |
| 5.2 The assumptions and basic properties . . . . .               | 174        |
| 5.3 Degenerate Schwartz functions I . . . . .                    | 178        |
| 5.4 Degenerate Schwartz functions II . . . . .                   | 181        |
| <b>6 Derivative of the Analytic Kernel</b>                       | <b>184</b> |
| 6.1 Decomposition of the derivative . . . . .                    | 184        |
| 6.2 Non-archimedean components . . . . .                         | 191        |
| 6.3 Archimedean components . . . . .                             | 196        |
| 6.4 Holomorphic projection . . . . .                             | 197        |
| 6.5 Holomorphic kernel function . . . . .                        | 202        |
| <b>7 Decomposition of the Geometric Kernel</b>                   | <b>206</b> |
| 7.1 Néron–Tate height . . . . .                                  | 207        |
| 7.2 Decomposition of the height series . . . . .                 | 216        |
| 7.3 Vanishing of the contribution of the Hodge classes . . . . . | 219        |
| 7.4 The goal of the next chapter . . . . .                       | 223        |
| <b>8 Local Heights of CM Points</b>                              | <b>230</b> |
| 8.1 Archimedean case . . . . .                                   | 230        |
| 8.2 Supersingular case . . . . .                                 | 233        |
| 8.3 Superspecial case . . . . .                                  | 239        |
| 8.4 Ordinary case . . . . .                                      | 244        |
| 8.5 The $j$ -part . . . . .                                      | 245        |
| <b>Bibliography</b>  | <b>251</b> |
| <b>Index</b>   | <b>255</b> |