Contents

Pı	refac	е		xv
1	Lin	ear reg	ression analysis	1
	1.1	Linear	regression assumptions	1
	1.2	Linear	regression estimation	3
		1.2.1	Residuals	7
		1.2.2	Outliers	7
	1.3	Linear	regression R-square	8
	1.4		regression standardization	8
	1.5	Exam	ple: Regression with one covariate	10
		1.5.1	Individual residuals and outliers	13
		1.5.2	Reporting results	15
	1.6	Multip	ble covariates	17
		1.6.1	Linear regression with two continuous covariates	17
			Interaction between two continuous covariates	18
		1.6.2	Linear regression with one binary and one continuous	
			covariate	20
			Interaction between a binary and a continuous covariate	e 22
	1.7	Examp	ple: Regression with two covariates	23
		1.7.1	Reporting results	25
	1.8	Examp	ble: Regression with an interaction	27
		1.8.1	Reporting results	29
			Presenting parameter estimates	30
			Presenting results graphically	30
	1.9	Specia	l topics	34
		1.9.1	Standardized coefficients greater than one	34
		1.9.2	Standardized coefficients differing in significance from	
			unstandardized coefficients	35
		1.9.3	Two-group regression analysis	36

		Example: Two-group regression analysis of an inter-	
		vention study	37
	1.9.4	Bringing covariates into the model	39
		Missing data on x	40
		Example: Bringing a covariate into the model for	
		the intervention example using a two-group	
		analysis	40
	1.9.5	The Bayesian Information Criterion (BIC) and the	
		Akaike Information Criterion (AIC)	44
	1.9.6	Heteroscedasticity modeling	45
		Example: Heteroscedasticity modeling of LSAY math	
		data	45
	1.9.7	Random coefficient regression	48
		Example: Random coefficient regression for LSAY	
		$\mathrm{math}\;\mathrm{data}\;\;.\;\ldots\;\ldots\;\ldots\;\ldots\;\ldots\;\ldots\;\ldots$	50
Mee		analysis	57
2.1	-	otypical mediation model	57
2.2		tion modeling techniques	59
	2.2.1	Estimation	61
		Indirect effect standard errors and confidence intervals	61
	2.2.2	Standardization for mediation models	63
	2.2.3	Model testing	65
2.3	Examp	ple: Sex discrimination	66
	2.3.1	Inspecting the data and reporting results	68
2.4	Examp	ple: Head circumference	72
	2.4.1	Reporting results	75
	2.4.2	The saturated model	80
2.5	Multip	le mediators	82
	2.5.1	Example: Parallel mediators for media influence	83
	2.5.2	Example: Sequential mediators of socioeconomic status	87
2.6	Moder	ated mediation	91
	2.6.1	Case 1 (xz) : Regression of y on x , m on x , both	
		moderated by z	91
	2.6.2	Case 2 (mz) : Regression of y on m moderated by z .	93
	2.6.3	Case 3 (mx) : Regression of y on m moderated by x .	95
	2.6.4	Combined moderation case	97
	2.6.5	Example: Case 1 moderated mediation in an interven-	
		tion of aggressive behavior in the classroom \ldots .	98

 $\mathbf{2}$

CONTENTS

			Testing significance of effects at specific moderator values	99
			Creating a plot with bootstrap confidence intervals for	00
			the effects at a range of moderator values	102
			Combination of significance of effect at specific mod-	
			erator values and plot of confidence intervals	
			using MODEL INDIRECT	106
		2.6.6	Example: Case 2 moderated mediation for work team	
			behavior	108
		2.6.7	Example: Case 3 moderated mediation of simulated	
			data	111
		2.6.8	Example: Combined moderated mediation for sex	
			discrimination	117
3	Spe	cial tor	pics in mediation analysis	121
-	3.1	-	Carlo simulation study of mediation	
		3.1.1	Example: Monte Carlo study of indirect effects	
	3.2		Carlo studies of moderation	
		3.2.1	Example: Moderation of the regression of m on x	
		3.2.2	Example: Moderation of the regression of y on m	
	3.3	Model	misspecification	
		3.3.1	Example: Omitted moderator	138
		3.3.2	Example: Omitted mediators	
		3.3.3	Example: Confounders	147
	3.4	Instrur	nental variable estimation	
		3.4.1	Example: IV estimation with mediator-outcome con-	
			founding	155
			Bias and coverage of IV estimation of the indirect effect	t156
			Comparing IV and ML standard errors and power	157
			IV estimation dependence on the size of the x, m	
			$\operatorname{correlation}$	158
			Comparison of IV and maximum-likelihood estima-	
			tion when the assumptions behind both ap-	
			proaches are violated $\ldots \ldots \ldots \ldots \ldots$	
	3.5		vity analysis	159
		3.5.1	Example: Sensitivity analysis for an experimental	
			study of sex discrimination in the workplace	162
			Example: Sensitivity analysis in a Monte Carlo study	165
	3.6	Multip	le-group mediation analysis	169

CONTENTS

		3.6.1	Relating multiple-group parameters to interaction pa-	
			rameters	0
		3.6.2	Modification indices	΄1
		3.6.3	Example: Two-group analysis of moderated media-	
			tion for sex discrimination	$^{\prime}2$
	3.7	Measu	rement errors and latent variables	6
		3.7.1	Measurement error in an independent variable 17	΄6
		3.7.2	Measurement error in a mediator	'8
		3.7.3	Example: Monte Carlo simulation study of measure-	
			ment error in the mediator	'9
		3.7.4	Known reliability	32
		3.7.5	Multiple indicators	32
			Reliability of a sum of indicators	33
			Structural equation modeling with a factor analysis	
			measurement model $\ldots \ldots \ldots \ldots \ldots \ldots 18$	35
4	Cau	ısal inf	erence for mediation 18	7
_	4.1		$1 \text{ assumptions} \dots 18$	38
	4.2		tial outcomes and counterfactuals	
		4.2.1	Example: Hypothetical potential outcome data 19) 0
	4.3	Basics	of counterfactually-defined effects	
		4.3.1	Example: Hypothetical mediation potential outcomes 19)3
	4.4	Direct	and indirect effects) 6
		4.4.1	Direct effects) 7
		4.4.2	Indirect effects) 8
		4.4.3	Total effect decomposition) 9
		4.4.4	Example: Hypothetical mediation data analysis 19	
	4.5		l effect formulas	
		4.5.1	Example: Effects in the simple mediation case 20)3
		4.5.2	Example: Effects with moderation of Y regressed on	
)4
		4.5.3	Example: Effects in the combined moderation case 20)6
		4.5.4	Example: Effects combining case 1 and case 2 moder-	
			ation	
		4.5.5		80
	4.6	Summ	ary	79
5	Cat			11
	5.1		concepts for categorical variables	
		5.1.1	Binary variables	14

CONTENTS

5.2	Binary	dependent variable	216
	5.2.1	Example: OLS, logistic, and probit regression of coal	
		miner respiratory problems	217
	5.2.2	Modeling with a logistic regression function	220
	5.2.3	Modeling with a probit regression function	222
	5.2.4	Estimation of the logistic and probit regressions	224
	5.2.5	Probability curve formulation versus a latent response	
		variable formulation	224
	5.2.6	R^2 and standardization $\ldots \ldots \ldots \ldots \ldots \ldots$	226
		R^2 for a binary outcome	227
		Standardization	227
	5.2.7	Example: Logistic and probit regression of British coal	
		miner data	228
		Logistic regression	228
		Probit regression	232
		Computation of estimated probabilities	232
		Comparing logistic and probit regression coefficients .	234
		Comparing the logistic and probit regression models	
		by BIC	234
	5.2.8	Logistic and probit regression with one binary and one	
		continuous x	234
	5.2.9	Logistic regression and adjusted odds ratios	235
	5.2.10	Example: Adjusted odds ratios for alcohol survey data	237
	5.2.11	Example: Adjusted odds ratios for educational achieve-	
		$ment \ data \ \ldots \ $	239
5.3	Ordina	d dependent variable	240
	5.3.1	Probability curve formulation of ordinal dependent	
		variable regression	240
	5.3.2	Latent response variable formulation of ordinal depen-	
		dent variable regression $\ldots \ldots \ldots \ldots \ldots \ldots$	243
	5.3.3	Example: Sample probits for drinking related to age	
			245
	5.3.4	Testing the parallel probability curve assumption	
		behind the ordinal regression model $\ldots \ldots \ldots$	245
	5.3.5	Example: Ordinal logistic regression of mental impair-	
		$ment \ldots \ldots$	247
		Estimated odds ratio	
		Estimated probabilities	
		Odds ratio with an interaction	250
5.4	Nomina	al dependent variable	252

		5.4.1	Example: Multinomial logistic regression of antisocial	
			behavior	253
6	Cou	nt dep	pendent variable	259
	6.1	Poisso	$n \bmod el \ldots \ldots$	259
	6.2	Poisso	n model with a random intercept $\ldots \ldots \ldots$	261
	6.3	Zero-i	nflated Poisson model	261
	6.4	0	ive binomial model	
	6.5	Zero-i	nflated negative binomial model	262
	6.6		part (hurdle) model with zero-truncation	
	6.7	Varyin	ng-exposure model	263
	6.8	Comp	aring models	264
	6.9	Exam	ple: Count regression of marital affairs	264
		6.9.1	Poisson, Poisson with a random intercept, and nega-	
			tive binomial models	. 266
			Negative binomial model	268
		6.9.2	Zero-inflated Poisson and zero-inflated negative bino-	
			mial models	. 269
		6.9.3	Two-part (hurdle) modeling	. 272
		6.9.4	Conclusion for marital affairs analyses	. 276
	6.10	Exam	ple: Poisson with varying exposure	. 276
7	Cen	sored	dependent variable	279
	7.1	Basic	concepts for a censored variable	. 279
	7.2	Censo	red-normal (tobit) regression	. 280
	7.3	Censo	red-inflated regression	. 282
	7.4	Sampl	e selection (Heckman) regression	. 283
		7.4.1	Example: Simulated sample selection data	. 285
	7.5	Two-p	part regression	. 288
	7.6	Exam	ple: Methods comparison on alcohol data	. 290
		7.6.1	Analysis results for the four models	. 293
			Loglikelihood and BIC comparisons of the four model	s 294
			Comparing the results for the censored-normal (tobit)	
			and censored-inflated models	. 295
			Comparing the results for the sample selection (Heck-	
			man) and two-part models \ldots	. 296
			Comparing the results for the censored-inflated and	
			two-part models	. 297

			Comparing the fit for estimated probabilities and means for the censored-inflated and two-part models	300
	7.7	Switch	ing regressions	
		7.7.1	Example: Monte Carlo simulation of switching regres-	
			sions	302
8	Me	diation	non-continuous variables	307
	8.1	Binary	outcome, continuous mediator	307
		8.1.1	A simple hypothetical mediation model	309
		8.1.2	Total, indirect, and direct effects in terms of differ-	
			ences in probabilities	310
		8.1.3	Causal effect formulas for a continuous M and a	
			binary Y	311
		8.1.4	Causal effect formulas applied to a simple mediation	
			model with a binary outcome and a continuous mediator	:314
		8.1.5	Causal effect formulas defined on the odds ratio scale	315
			Odds ratio effects assuming a rare outcome	315
		8.1.6	Causal effects with multiple mediators and a binary	
			outcome	
		8.1.7	Example: Intention to use cigarettes	
			Probit regression	
			Logistic regression	
		8.1.8	Example: HPV vaccination trial	
			No intervention-mediator interaction	
			Intervention-mediator interaction	
			Analysis results	
	8.2		outcome, continuous mediator	
		8.2.1	Causal effect formulas for a count outcome	
		8.2.2	Example: Aggressive behavior and school removal	
			Estimated count probabilities	
	8.3		art outcome, continuous mediator	
		8.3.1		337
		8.3.2	Example: Two-part mediation analysis of economic	000
			stress data	
			Causal effects for two-part modeling	
			Causal effects for regular modeling with $\log y$	
			Causal effects for regular modeling without $\log y$	
	8.4		and ordinal mediator	346
		8.4.1	Causal effect formulas for a binary mediator	346

		8.4.2	Ordinal mediator
		8.4.3	Latent response variable mediator
		8.4.4	Estimation
		8.4.5	Example: Hypothetical data from the potential out-
			come example with a binary mediator and a continu-
			ous outcome
		8.4.6	Example: Ordinal mediator for intention to use cigarettes 35
		8.4.7	Example: Pearl's artificial 2 x 2 x 2 example 357
	8.5	Nomir	nal mediator
		8.5.1	Causal effect formulas for a nominal mediator 361
		8.5.2	Estimation
		8.5.3	Example: Hypothetical data with a nominal mediator
			and a binary outcome
	8.6	Media	tor with measurement error
		8.6.1	Example: A Monte Carlo simulation study for a
			mediator measured with error
		8.6.2	Example: An intervention study of aggressive behav-
			ior in the classroom and juvenile court record 374
9	Bay	esian a	analysis 381
•	9.1		likelihood, and posterior
		9.1.1	
			Posterior distribution for the mean of a normal distri-
			Posterior distribution for the mean of a normal distribution
		9.1.1	Posterior distribution for the mean of a normal distributionbution
	9.2	9.1.1 9.1.2 9.1.3	Posterior distribution for the mean of a normal distributionbutionTypes of priorsNon-normality of parameter distributions386
	9.2	9.1.1 9.1.2 9.1.3	Posterior distribution for the mean of a normal distributionbution
	9.2	9.1.1 9.1.2 9.1.3 Marko	Posterior distribution for the mean of a normal distribution bution 383 Types of priors 385 Non-normality of parameter distributions 386 ov Chain Monte Carlo (MCMC) 387
	9.2	9.1.1 9.1.2 9.1.3 Marko	Posterior distribution for the mean of a normal distribution
	9.2	9.1.1 9.1.2 9.1.3 Marko	Posterior distribution for the mean of a normal distribution 383 Types of priors 385 Non-normality of parameter distributions 386 ov Chain Monte Carlo (MCMC) 387 Example: Bayesian estimation of a mean with missing 388 data for LSAY math 388
	9.2	9.1.1 9.1.2 9.1.3 Marko 9.2.1	Posterior distribution for the mean of a normal distribution 383 Types of priors 385 Non-normality of parameter distributions 386 ov Chain Monte Carlo (MCMC) 387 Example: Bayesian estimation of a mean with missing 388 Input for Bayesian analysis 387
	9.2	9.1.1 9.1.2 9.1.3 Marko 9.2.1	Posterior distribution for the mean of a normal distribution 383 Types of priors 385 Non-normality of parameter distributions 386 ov Chain Monte Carlo (MCMC) 387 Example: Bayesian estimation of a mean with missing 388 Input for Bayesian analysis 391 Plots 393
	9.2	9.1.1 9.1.2 9.1.3 Marko 9.2.1	Posterior distribution for the mean of a normal distribution 383 Types of priors 385 Non-normality of parameter distributions 386 ov Chain Monte Carlo (MCMC) 387 Example: Bayesian estimation of a mean with missing 388 Input for Bayesian analysis 391 Plots 393 Trace plot 393
	9.2	9.1.1 9.1.2 9.1.3 Marko 9.2.1	Posterior distribution for the mean of a normal distributionbutionTypes of priorsTypes of priorsNon-normality of parameter distributionsW Chain Monte Carlo (MCMC)Example: Bayesian estimation of a mean with missingdata for LSAY mathInput for Bayesian analysisPlotsTrace plotAutocorrelation plot395
	9.2 9.3	 9.1.1 9.1.2 9.1.3 Marko 9.2.1 9.2.2 	Posterior distribution for the mean of a normal distribution 383 Types of priors 385 Non-normality of parameter distributions 386 ov Chain Monte Carlo (MCMC) 387 Example: Bayesian estimation of a mean with missing 388 data for LSAY math 388 Input for Bayesian analysis 391 Plots 393 Trace plot 395 Posterior distribution plot 395 Posterior distribution plot 395 Overgence checking 395
	9.3 9.4	 9.1.1 9.1.2 9.1.3 Marko 9.2.1 9.2.2 9.2.3 Model Bayes 	Posterior distribution for the mean of a normal distributionbutionTypes of priorsTypes of priorsNon-normality of parameter distributionsW Chain Monte Carlo (MCMC)Example: Bayesian estimation of a mean with missingdata for LSAY mathInput for Bayesian analysisPlotsTrace plotAutocorrelation plot995Posterior distribution plot995fit4000versus ML intervals
	9.3	 9.1.1 9.1.2 9.1.3 Marko 9.2.1 9.2.2 9.2.3 Model Bayes 	Posterior distribution for the mean of a normal distribution 383 Types of priors 383 Non-normality of parameter distributions 386 ow Chain Monte Carlo (MCMC) 387 Example: Bayesian estimation of a mean with missing 388 data for LSAY math 388 Input for Bayesian analysis 391 Plots 393 Autocorrelation plot 395 Posterior distribution plot 395 fit 400 versus ML intervals 400 ple: Mediation model for media influence 404
	9.3 9.4	 9.1.1 9.1.2 9.1.3 Marko 9.2.1 9.2.2 9.2.3 Model Bayes 	Posterior distribution for the mean of a normal distributionbution383Types of priors385Non-normality of parameter distributions386ov Chain Monte Carlo (MCMC)387Example: Bayesian estimation of a mean with missingdata for LSAY math388Input for Bayesian analysis391Plots393Trace plot393Autocorrelation plot393Convergence checking395fit400versus ML intervals402ple: Mediation model for media influence404The Potential Scale Reduction (PSR) convergence404
	9.3 9.4	 9.1.1 9.1.2 9.1.3 Marko 9.2.1 9.2.2 9.2.3 Model Bayes 	Posterior distribution for the mean of a normal distribution 383 Types of priors 383 Non-normality of parameter distributions 386 ow Chain Monte Carlo (MCMC) 387 Example: Bayesian estimation of a mean with missing 388 data for LSAY math 388 Input for Bayesian analysis 391 Plots 393 Autocorrelation plot 395 Posterior distribution plot 395 fit 400 versus ML intervals 400 ple: Mediation model for media influence 404

			Autocorrelation plot)6
			Inspecting model fit and parameter estimates 40)8
	9.6	Examp	ble: Mediation model for firefighter data	13
		9.6.1	Non-informative priors	13
		9.6.2	Informative priors	13
	9.7	Exam	ble: Model testing of direct effects	16
	9.8	Examp	ple: High school dropout and missing data 4	18
		9.8.1	Missing data on the mediator $(n = 2, 213)$ 4	18
			Bayesian analysis	19
			Maximum-likelihood analysis	22
		9.8.2	Missing data on the control variables $(n = 2, 898)$ 42	23
			Assuming normality for all covariates	24
			Acknowledging that some control variables are binary 4.	25
10	Mis	sing da	ata 42	27
	10.1	Exam	ple: Missing data information	27
			$\mathbf{R}, \mathbf{MAR}, \mathbf{and} \mathbf{NMAR} \dots \dots$	
			MCAR: Missing completely at random	
		10.2.2	MAR: Missing at random	30
		10.2.3	NMAR: Not missing at random	31
	10.3	MAR	for bivariate normal variables $(H_1 \text{ case}) \ldots \ldots \ldots 43$	31
		10.3.1	Listwise versus ML	32
		10.3.2	Maximum-likelihood estimation in the bivariate case	
			with missing on one variable	34
		10.3.3	The EM algorithm	36
		10.3.4	Multiple imputation	38
		10.3.5	Example: Estimating sample statistics for interven-	
			tion data	
	10.4		for regression $(H_0 \text{ case})$	
			Missing data and selection on x or y 4	
			Regression analysis with missing data 4	
			Technical aspects of ML assuming MAR 4	
			Example: MAR simulated data analysis 4	
		10.4.5	Missing data correlates	56
			Example: Simulation study with missing data corre-	
			late of missing on y	
	10.5		. <i>.</i>	64
		10.5.1	Example: Simulated NMAR data with missing influ-	<u> </u>
			enced by the latent outcome	65

10.5.2 Example: Selection modeling versus ML assuming
MAR when MAR holds
10.6 Example: Comparing missing data methods
10.7 Missing data on covariates
10.7.1 Example: Simulation study of missing on binary
covariates
Inputs for generating and analyzing data assuming MAR476
Inputs for generating and analyzing data under NMAR 480
Simulation results
Appendices 489
A Covariance algebra 491
A.1 Definition of an expectation
A.1 Definition of an expectation 491 A.1.1 Rules for an expectation 492
A.1 Definition of an expectation 491 A.1.1 Rules for an expectation 492 A.2 Definition of covariance and variance 492
A.1 Definition of an expectation 491 A.1.1 Rules for an expectation 492 A.2 Definition of covariance and variance 492 A.2.1 Rules for variance 493
A.1 Definition of an expectation 491 A.1.1 Rules for an expectation 492 A.2 Definition of covariance and variance 492 A.2.1 Rules for variance 493 A.3 Functions of random variables 493
A.1 Definition of an expectation 491 A.1.1 Rules for an expectation 492 A.2 Definition of covariance and variance 492 A.2.1 Rules for variance 493 A.3 Functions of random variables 493
A.1 Definition of an expectation 491 A.1.1 Rules for an expectation 492 A.2 Definition of covariance and variance 492 A.2.1 Rules for variance 493 A.3 Functions of random variables 493 A.4 Example: Covariance algebra rules applied to linear regression 493
A.1 Definition of an expectation 491 A.1.1 Rules for an expectation 492 A.2 Definition of covariance and variance 492 A.2.1 Rules for variance 493 A.3 Functions of random variables 493 A.4 Example: Covariance algebra rules applied to linear regression 493 A.5 Example: Derivation of the slope attenuation 494