

McGRAW-HILL BOOK COMPANY

Auckland Bogota Guatemala Hamburg Lisbon  
London Madrid Mexico New Delhi Panama Paris San Juan  
São Paulo Singapore Sydney Tokyo

**WALTER RUDIN**

*Professor of Mathematics  
University of Wisconsin—Madison*

# Principles of Mathematical Analysis

**THIRD EDITION**

# CONTENTS

<b>Preface</b>	ix
<b>Chapter 1 The Real and Complex Number Systems</b>	<b>1</b>
Introduction	1
Ordered Sets	3
Fields	5
The Real Field	8
The Extended Real Number System	11
The Complex Field	12
Euclidean Spaces	16
Appendix	17
Exercises	21
<b>Chapter 2 Basic Topology</b>	<b>24</b>
Finite, Countable, and Uncountable Sets	24
Metric Spaces	30
Compact Sets	36
Perfect Sets	41

	Connected Sets	42
	Exercises	43
<b>Chapter 3</b>	<b>Numerical Sequences and Series</b>	<b>47</b>
	Convergent Sequences	47
	Subsequences	51
	Cauchy Sequences	52
	Upper and Lower Limits	55
	Some Special Sequences	57
	Series	58
	Series of Nonnegative Terms	61
	The Number $e$	63
	The Root and Ratio Tests	65
	Power Series	69
	Summation by Parts	70
	Absolute Convergence	71
	Addition and Multiplication of Series	72
	Rearrangements	75
	Exercises	78
<b>Chapter 4</b>	<b>Continuity</b>	<b>83</b>
	Limits of Functions	83
	Continuous Functions	85
	Continuity and Compactness	89
	Continuity and Connectedness	93
	Discontinuities	94
	Monotonic Functions	95
	Infinite Limits and Limits at Infinity	97
	Exercises	98
<b>Chapter 5</b>	<b>Differentiation</b>	<b>103</b>
	The Derivative of a Real Function	103
	Mean Value Theorems	107
	The Continuity of Derivatives	108
	L'Hospital's Rule	109
	Derivatives of Higher Order	110
	Taylor's Theorem	110
	Differentiation of Vector-valued Functions	111
	Exercises	114

<b>Chapter 6</b>	<b>The Riemann-Stieltjes Integral</b>	<b>120</b>
	Definition and Existence of the Integral	120
	Properties of the Integral	128
	Integration and Differentiation	133
	Integration of Vector-valued Functions	135
	Rectifiable Curves	136
	Exercises	138
<b>Chapter 7</b>	<b>Sequences and Series of Functions</b>	<b>143</b>
	Discussion of Main Problem	143
	Uniform Convergence	147
	Uniform Convergence and Continuity	149
	Uniform Convergence and Integration	151
	Uniform Convergence and Differentiation	152
	Equicontinuous Families of Functions	154
	The Stone-Weierstrass Theorem	159
	Exercises	165
<b>Chapter 8</b>	<b>Some Special Functions</b>	<b>172</b>
	Power Series	172
	The Exponential and Logarithmic Functions	178
	The Trigonometric Functions	182
	The Algebraic Completeness of the Complex Field	184
	Fourier Series	185
	The Gamma Function	192
	Exercises	196
<b>Chapter 9</b>	<b>Functions of Several Variables</b>	<b>204</b>
	Linear Transformations	204
	Differentiation	211
	The Contraction Principle	220
	The Inverse Function Theorem	221
	The Implicit Function Theorem	223
	The Rank Theorem	228
	Determinants	231
	Derivatives of Higher Order	235
	Differentiation of Integrals	236
	Exercises	239
<b>Chapter 10</b>	<b>Integration of Differential Forms</b>	<b>245</b>
	Integration	245

Primitive Mappings	248
Partitions of Unity	251
Change of Variables	252
Differential Forms	253
Simplexes and Chains	256
Stokes' Theorem	273
Closed Forms and Exact Forms	275
Vector Analysis	280
Exercises	288
<b>Chapter 11 The Lebesgue Theory</b>	<b>300</b>
Set Functions	300
Construction of the Lebesgue Measure	302
Measure Spaces	310
Measurable Functions	310
Simple Functions	313
Integration	314
Comparison with the Riemann Integral	322
Integration of Complex Functions	325
Functions of Class $\mathcal{L}^2$	325
Exercises	332
<b>Bibliography</b>	<b>335</b>
<b>List of Special Symbols</b>	<b>337</b>
<b>Index</b>	<b>339</b>