Contents

Preface		page xiii	
Intro	1		
0	Polytopes and Linear Programming	9	
0.1	Finite Systems of Linear Inequalities	9	
0.2	Linear-Programming Duality	14	
0.3	Basic Solutions and the Primal Simplex Method	21	
0.4	Sensitivity Analysis	27	
0.5	Polytopes	29	
0.6	Lagrangian Relaxation	35	
0.7	The Dual Simplex Method	40	
0.8	Totally Unimodular Matrices, Graphs, and Digraphs	41	
0.9	Further Study	47	
1	Matroids and the Greedy Algorithm	49	
1.1	Independence Axioms and Examples of Matroids	49	
1.2	Circuit Properties	51	
1.3	Representations	53	
1.4	The Greedy Algorithm	56	
1.5	Rank Properties	60	
1.6	Duality	63	
1.7	The Matroid Polytope	66	
1.8	Further Study	73	
2	Minimum-Weight Dipaths	75	
2.1	No Negative-Weight Cycles	76	
2.2	All-Pairs Minimum-Weight Dipaths	78	

х	Contents	
23	Nonnegative Weights	78
2.5	No Dicycles and Knapsack Programs	81
2.5	Further Study	82
•	Maturid Tutorspotion	84
3	Applications	84
3.1	Applications	
3.2	An Encient Calumanty Matolu-mersection	89
, ,	Argonium and Consequences	101
3.3 7 1	All Enclent Maximum-weight Mationa mersection regonant	103
3.4 25	Further Study	106
5.5	Future Study	
4	Matching	107
4.1	Augmenting Paths and Matroids	107
4.2	The Matching Polytope	109
4.3	Duality and a Maximum-Cardinality Matching Algorithm	113
4.4	Kuhn's Algorithm for the Assignment Problem	121
4.5	Applications of Weighted Matching	126
4.6	Further Study	137
5	Flows and Cuts	138
5.1	Source-Sink Flows and Cuts	138
5.2	An Efficient Maximum-Flow Algorithm and Consequences	140
5.3	Undirected Cuts	147
5.4	Further Study	150
6	Cutting Planes	151
6.1	Generic Cutting-Plane Method	151
6.2	Chvátal–Gomory Cutting Planes	152
6.3	Gomory Cutting Planes	156
6.4	Tightening a Constraint	167
6.5	Constraint Generation for Combinatorial-Optimization	
	Problems	171
6.6	Further Study	176
7	Branch-&-Bound	177
7.1	Branch-&-Bound Using Linear-Programming Relaxation	179
7.2	Knapsack Programs and Group Relaxation	184
7.3	Branch-&-Bound for Optimal-Weight Hamiltonian Tour	188
		100

Contents

	Contents	xi
7.4	Maximum-Entropy Sampling and Branch-&-Bound	191
7.5	Further Study	193
8	Optimizing Submodular Functions	194
8.1	Minimizing Submodular Functions	194
8.2	Minimizing Submodular Functions Over Odd Sets	197
8.3	Maximizing Submodular Functions	200
8.4	Further Study	201
Appendix: Notation and Terminology		203
A.1	Sets	203
A.2	Algebra	203
A.3	Graphs	204
A.4	Digraphs	205
Refe	References	
	Background Reading	207
	Further Reading	207
Inde	Indexes	
	Examples	209
	Exercises	209
	Problems	209
	Results	210
	Algorithms	211