Contents

Advice to the student on using this book xi

Functions, limits, and continuity 1

1 Functions 2
1.1 Introduction 2
1.2 Functions and their graphs 8
1.3 Polynomials 19
1.4 Rational functions 31
1.5 Inverse functions 35

2 Elementary functions used in calculus 43
2.1 Exponential and logarithmic functions 43
2.2 Trigonometric functions 47
2.3 Inverse trigonometric functions 53
2.4 Hyperbolic functions 57
2.5 Inverse hyperbolic functions 59

3 Limits and continuity 63
3.1 Limits 67
3.2 One-sided limits 77
3.3 Infinite limits 81
3.4 Continuous functions 85
3.5 Continuous functions on closed intervals 95
3.6 Proofs 102

Derivatives 105

4 Differentiation 106
4.1 Tangency 106
4.2 Differentiability 112
4.3 Derivative function 120
4.4 Special derivatives 126
4.5 Rectilinear motion and velocity 133
4.6 Approximations 138
4.7 Higher derivatives 141
4.8 Acceleration 143

5 Differentiation rules 148
5.1 Product and quotient rules 148
5.2 Chain rule and implicit differentiation 155
5.3 Rates of change 165
5.4 Derivatives of inverse functions 169

6 Extremum problems 179
6.1 Terminology 180
6.2 Necessary condition for a local extremum 182
6.3 First-derivative test 193
6.4 Second-derivative test 198
6.5 Optimal inventory 203
6.6 Convexity 206
6.7 Analysis of graphs 217
6.8 Proofs 222

7 Mean value theorem 224
7.1 Mean value theorem 225
7.2 Rule of l'Hospital 232
7.3 Taylor theorem 243
7.4 Antiderivatives 247
7.5 Iterative methods 251
7.6 Newton method 258
7.7 Fixed points 268
7.8 Proofs 272

Integrals 279

8 Definite integrals 280
8.1 Introduction 280
8.2 Riemann sums 283
8.3 Definite integral 291
8.4 Numerical integration: trapezoid method 308
8.5 Numerical integration: Simpson method 312
8.6 Proofs 319

9 Fundamental theorem of calculus 322
9.1 Indefinite integral 322
9.2 Position and distance from velocity 333
9.3 Fundamental theorem of calculus 334
9.4 List of integrals 341
9.5 Proofs 343

10 Integration techniques 344
10.1 Changing variables 344
10.2 Integration by parts 360
10.3 Rational functions 368

10.4 Improper integrals: infinite intervals 373
10.5 Improper integrals: unbounded integrands 381

11 Applications of integrals 384
11.1 Area 384
11.2 Area by polar coordinates 388
11.3 Arc length 391
11.4 Volume 394
11.5 Solids of revolution: volume 398
11.6 Solids of revolution: surface area 402
11.7 Moments and centroids 404
11.8 Centroids of three-dimensional bodies 412
11.9 Work 416
11.10 Hydrostatic force 423

Series and approximations 427

12 Sequences and series 428
12.1 Sequences and convergence 428
12.2 Series 438
12.3 Convergence criteria for series 443
12.4 Proofs 456

13 Series expansions and approximations 460
13.1 Series of functions 460
13.2 Power series 462
13.3 Differentiation of power series 472
13.4 Taylor series 480
13.5 Lagrange interpolation 490
13.6 Proofs 494

Appendixes 501

A Introduction to MACSYMA 502
A.1 MACSYMA inputs and outputs 502
A.2 Getting on-line help 503
A.3 Expressions 503
A.4 Constants 504
A.5 Numbers 505
A.6 Assignments 506
A.7 Equations 508
A.8 Functions 509
A.9 Lists 510
A.10 Expanding expressions 510
A.11 Simplifying expressions 512
A.12 Factoring expressions 513
A.13 Making substitutions 515
A.14 Extracting parts of an expression 516

A.15 Trigonometric functions 517
A.16 A simple program 519
A.17 Plotting 521

B Numbers 527
B.1 Arithmetic operations 527
B.2 Real numbers 529
B.3 Absolute value 535
B.4 Equations and inequalities 537
B.5 Two fundamental properties of real numbers 538
B.6 Complex numbers 541

C Analytical geometry 545
C.1 Plane $\mathbf{R}^2$ with Cartesian coordinates 545
C.2 Lines 549
C.3 Circles 558
C.4 Sine, cosine, and tangent 562
C.5 Polar coordinates 569

D Conic sections 575
D.1 Conic sections 575
D.2 Circle 577
D.3 Parabola 578
D.4 Ellipse 582
D.5 Hyperbola 587
D.6 The general quadratic $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$ 592

Subject index 599