

Contents

Acknowledgments ix

Introduction xi

1. Background 1

Mathematical Certainty 2

Boole's Logic 4

Mathematical Logic 5

Securing the Foundations of Mathematics 7

Hilbert's Approach 8

Gödel's Results 10

Turing's Results 11

2. Some Undecidable Decision Problems 15

Emil Post 16

Post's Correspondence Problem 17

Hilbert's Tenth Problem 22

The Halting Problem 24

Back to Turing at Cambridge 24

3. Finite Automata 25

Introduction 25

Finite Automata 27

Our First Machine 27

Alphabets and Languages 29

Finite Automata and Answering Questions 31

Omitting Traps from Diagrams 33

Some Basic Facts 35

Regular Expressions 37

Limitations of Finite Automata 41

Tapes and Configurations 43

Connection to the Correspondence Problem 44

4. Turing Machines 47

Examples of Turing Machines 52

Computable Functions and Calculations 59

Church-Turing Thesis 61

Computational Power	63
Machines That Don't Halt	67
5. Other Systems for Computation	69
The Lambda Calculus	72
Tag Systems	79
One-Dimensional Cellular Automata	82
6. Encodings and the Universal Machine	87
A Method of Encoding Finite Automata	88
Universal Machines	91
Construction of Universal Machines	93
Modern Computers Are Universal Machines	95
Von Neumann Architecture	97
Random Access Machines	98
RAMs Can Be Emulated by Turing Machines	101
Other Universal Machines	103
What Happens When We Input (M) into M ?	104
7. Undecidable Problems	107
Proof by Contradiction	108
Russell's Barber	111
Finite Automata That Do Not Accept Their Encodings	113
Turing Machines That Do Not Accept Their Encodings	114
<i>Does a Turing Machine Diverge on Its Encoding?</i> Is Undecidable	116
The Acceptance, Halting, and Blank Tape Problems	117
An Uncomputable Function	119
Turing's Approach	120
8. Cantor's Diagonalization Arguments	123
Georg Cantor 1845–1918	123
Cardinality	124
Subsets of the Rationals That Have the Same Cardinality	126
Hilbert's Hotel	129
Subtraction Is Not Well-Defined	131
General Diagonal Argument	131
The Cardinality of the Real Numbers	135
The Diagonal Argument	138

The Continuum Hypothesis	139
The Cardinality of Computations	140
Computable Numbers	141
A Non-Computable Number	142
There Is a Countable Number of Computable Numbers	143
Computable Numbers Are Not Effectively Enumerable	143

9. Turing's Legacy 147

Turing at Princeton	148
Second World War	150
Development of Computers in the 1940s	153
The Turing Test	157
Downfall	159
Apology and Pardon	161

Further Reading 163

Notes 167

Bibliography 183

Index 187