Contents

Preface v

1 Measures and Transforms 1
1.1 Measures 1
1.2 Fourier Transform 5
1.3 The Wavelet Transform 7
1.4 Borel Transform 16
1.5 Gesztesy–Krein–Simon ξ Function 24
1.6 Notes 25

2 Selfadjointness and Spectrum 29
2.1 Selfadjointness 29
2.1.1 Linear Operators and Their Inverses 29
2.1.2 Closed Operators 30
2.1.3 Adjoint and Selfadjoint Operators 32
2.1.4 Sums of Linear Operators 34
2.1.5 Sesquilinear Forms 35
2.2 Spectrum and Resolvent Sets 37
2.3 Spectral Theorem 40
2.4 Spectral Measures and Spectrum 43
2.5 Spectral Theorem in the Hahn–Hellinger Form 45
2.6 Components of the Spectrum 49
2.7 Characterization of the States in Spectral Subspaces 53
2.8 Notes 56

3 Criteria for Identifying the Spectrum 59
3.1 Borel Transform 59
3.2 Fourier Transform 68
3.3 Wavelet Transform 69
3.4 Eigenfunctions 70
3.5 Commutators 72

3.6 Criteria Using Scattering Theory 80
3.6.1 Wave Operators 81
3.6.2 Stability of the Absolutely Continuous Spectra 95
3.7 Notes 104

4 Operators of Interest 111
4.1 Unperturbed Operators 111
4.1.1 Laplacians 112
4.1.2 Unperturbed Semigroups and Their Kernels 119
4.1.3 Associated Processes 120
4.1.4 Regular Dirichlet Forms, Capacities and Equilibrium Potentials 121
4.2 Perturbed Operators 125
4.2.1 Deterministic Potentials 125
4.2.2 Random Potentials 133
4.2.3 Singular Perturbations 135
4.3 Notes 142

5 Applications 153
5.1 Borel Transforms 153
5.1.1 Kotani Theory 153
5.1.2 Aizenman–Molchanov Method 160
5.1.3 Bethe Lattice 172
5.1.4 Jaksić–Last Theorem 181
5.2 Scattering 183
5.2.1 Decaying Random Potentials 183
5.2.2 Obstacles and Potentials 187
5.3 Notes 196

References 203

Index 215