

Contents

Preface	v
Introduction	xiii
Contributors and speakers	xix
Lectures	xxvii

Chapter 1 The Hierarchy of Climate Models..... 1

A gallery of simple models from climate physics

DIRK OLBERS.....	3
1 Introduction	3
2 Fluid dynamics and thermodynamics.....	6
3 Reduced physics equations	14
4 Integrated models.....	26
5 Low-order models.....	44
References	58

Simple climate models

KLAUS FRAEDRICH.....	65
1 Climate and climate modelling	65
2 Zero-dimensional energy balance climate model	68
3 Quasi-geostrophic two-layer atmosphere in a channel.....	77
4 Reduced gravity ocean in a square basin	90
5 Summary and outlook	97
References	98

Complex climate models – tools for studying the origin of stochasticity in the climate system

JIN-SONG VON STORCH.....	101
1 Introduction.....	101
2 Origin of complexity.....	104
3 Cosequence of complexity.....	111
4 Concluding remarks	114
References	115

Some mathematical aspects of the GCMs

ROGER TEMAM.....	117
1 Introduction.....	117
2 Hierarchy of PDEs in the GCMs.....	118
3 The PEs and PEV ² s of the ocean.....	119

4	The PEs and PEV ² s of the atmosphere	129
5	Coupled atmosphere-ocean (CAO) models	134
	References	136

Chapter 2 The Emergence of Randomness: Chaos, Averaging, Limit Theorems 139

Hasselmann’s program revisited: the analysis of stochasticity in deterministic climate models

LUDWIG ARNOLD.....	141
1 Introduction.....	141
2 Stochasticity in deterministic climate models with two separate time scales.....	143
3 The method of averaging	146
4 Normal deviations from the averaged system: the central limit theorem.....	148
5 Large deviations from the averaged system.....	151
6 Extensions of Hasselmann’s program comments.....	154
References	155

Thermodynamic formalism, large deviation, and multifractals

MANFRED DENKER AND MARC KESSEBÖHMER.....	159
1 Expanding dynamical systems	159
2 Pointwise dimension	163
3 Multifractal formalism	165
4 Local large deviation	166
References	168
Summary	168

Averaging and climate models

YURI KIFER.....	171
1 Introduction.....	171
2 The averaging setup	172
3 Fully coupled systems	177
4 Appendix A: hyperbolicity	182
5 Appendix B: Proof of Theorem 3.1.....	183
6 Appendix B: Proof of Theorem 3.2.....	185
References	187

**Dynamical systems with time scale separation:
averaging, stochastic modelling, and central
limit theorems**

CHRISTIAN RÖDENBECK, CHRISTIAN BECK
AND HOLGER KANTZ..... 189

- 1 Introduction..... 189
- 2 Average skill of an averaged model..... 190
- 3 Stochastic modelling..... 194
- 4 Central limit theorems and their limits..... 201
- Conclusion..... 206
- Appendix: Remarks on the numerical implementation..... 207
- References..... 208

**Chapter 3 Tools and Methods: SDE, Dynamical Systems,
SPDE, Multiscale Techniques..... 211**

Energy balance models – viewed from stochastic dynamics
PETER IMKELLER..... 213

- 1 Introduction..... 213
- 2 The paradigm of stochastic resonance..... 215
- 3 Deterministic energy balance models..... 219
- 4 Stochastic extensions of EBM..... 228
- 5 Stochastic resonance: Freidlin’s approach..... 229
- References..... 236

**Exponential stability of the quasigeostrophic
equation under random perturbations**
JINQIAO DUAN, PETER E. KLOEDEN AND BJÖRN SCHMALFUSS..... 241

- 1 Introduction..... 241
- 2 Preliminaries..... 243
- 3 Transformation of the quasigeostrophic equation..... 247
- 4 The stationary solution..... 249
- 5 Discussion..... 255
- References..... 255

A mini course on stochastic partial differential equations
JERZY ZABCZYK..... 257

- 1 Introduction..... 257
- 2 Cauchy problem and semigroups..... 258
- 3 Infinite dimensional Wiener processes..... 263
- 4 Stochastic integration..... 265

5	First order stochastic equations	266
6	Heat equation with space-time white noise	269
7	Stationary solutions of a wave equation	271
8	Nonlinear stochastic pdes	275
9	Appendix	279
	References	283
Hasselmann's stochastic climate model viewed from a statistical mechanics perspective		
	PETER MÜLLER	285
1	Introduction	285
2	The microscopic description	287
3	The mesoscopic description	288
4	A derivation of the Langevin equation	289
5	The macroscopic description	292
6	Statistical mechanics	293
7	Discussion	294
	References	294
Chapter 4 Reduced Stochastic Models and Particular Techniques		
		297
Constrained stochastic forcing		
	JOSEPH EGGER	299
1	Introduction	299
2	Charney-DeVore model	300
3	Discussion	306
	References	307
Stochastic resonance and noise-induced phase coherence		
	JAN A. FREUND, ALEXANDER NEIMAN AND LUTZ SCHIMANSKY-GEIER	309
1	Introduction	309
2	Stochastic resonance in the framework of synchronization phenomena	316
3	Conclusions	322
	References	322
Stochastic confinement of Rossby waves by fluctuating eastward flows		
	ADAM HUGH MONAHAN, LIONEL PANDOLFO AND PETER IMKELLER	325

1	Introduction	325
2	Spectral model	327
3	Superrotation flow	331
4	Interpretation	335
5	Conclusions	340
	References	341
Some mathematical remarks concerning the localization of planetary waves in a stochastic background flow		
PETER IMKELLER, ADAM HUGH MONAHAN AND LIONEL PANDOLFO		
		345
1	Introduction	345
2	Some remarks concerning path properties of R	347
3	Transformation into Sturm-Liouville problems	349
4	Critical lines for $\mu = 0$	353
5	Critical lines for general μ	355
6	The spectrum of L	358
7	The spectrum of K	364
	References	367
Rossby waves in a stochastically fluctuating medium		
PRASHANT SARDESHMUKH, CÉCILE PENLAND AND MATTHEW NEWMAN		
		369
1	Introduction	369
2	The stochastic differential equations	372
	Results	379
	Summary and discussion	382
	Appendix A	382
	References	383
Passive tracer transport in stochastic flows		
W.A. WOYCZYŃSKI		
		385
1	Introduction	385
2	Lagrangian vs. Eulerian picture	385
3	Slowly varying spatial variables	388
4	Richardson function of an advected scalar	389
5	Statistical topography of passive tracer fields	390
6	Other directions	393
	References	396