

# Contents

<b>Preface</b>	<b>IX</b>
<b>Introduction</b>	<b>1</b>
<b>1 Function Spaces, Linear Operators and Green's Functions</b>	<b>5</b>
1.1 Function Spaces . . . . .	5
1.2 Orthonormal System of Functions . . . . .	7
1.3 Linear Operators . . . . .	8
1.4 Eigenvalues and Eigenfunctions . . . . .	11
1.5 The Fredholm Alternative . . . . .	12
1.6 Self-adjoint Operators . . . . .	15
1.7 Green's Functions for Differential Equations . . . . .	16
1.8 Review of Complex Analysis . . . . .	21
1.9 Review of Fourier Transform . . . . .	28
<b>2 Integral Equations and Green's Functions</b>	<b>33</b>
2.1 Introduction to Integral Equations . . . . .	33
2.2 Relationship of Integral Equations with Differential Equations and Green's Functions . . . . .	39
2.3 Sturm–Liouville System . . . . .	44
2.4 Green's Function for Time-Dependent Scattering Problem . . . . .	48
2.5 Lippmann–Schwinger Equation . . . . .	52
2.6 Problems for Chapter 2 . . . . .	57
<b>3 Integral Equations of Volterra Type</b>	<b>63</b>
3.1 Iterative Solution to Volterra Integral Equation of the Second Kind . . . . .	63
3.2 Solvable cases of Volterra Integral Equation . . . . .	66
3.3 Problems for Chapter 3 . . . . .	71
<b>4 Integral Equations of the Fredholm Type</b>	<b>75</b>
4.1 Iterative Solution to the Fredholm Integral Equation of the Second Kind . . . . .	75
4.2 Resolvent Kernel . . . . .	78
4.3 Pincherle–Goursat Kernel . . . . .	81
4.4 Fredholm Theory for a Bounded Kernel . . . . .	86
4.5 Solvable Example . . . . .	93

4.6	Fredholm Integral Equation with a Translation Kernel . . . . .	95
4.7	System of Fredholm Integral Equations of the Second Kind . . . . .	100
4.8	Problems for Chapter 4 . . . . .	101
<b>5</b>	<b>Hilbert–Schmidt Theory of Symmetric Kernel . . . . .</b>	<b>109</b>
5.1	Real and Symmetric Matrix . . . . .	109
5.2	Real and Symmetric Kernel . . . . .	111
5.3	Bounds on the Eigenvalues . . . . .	122
5.4	Rayleigh Quotient . . . . .	126
5.5	Completeness of Sturm–Liouville Eigenfunctions . . . . .	129
5.6	Generalization of Hilbert–Schmidt Theory . . . . .	131
5.7	Generalization of Sturm–Liouville System . . . . .	138
5.8	Problems for Chapter 5 . . . . .	144
<b>6</b>	<b>Singular Integral Equations of Cauchy Type . . . . .</b>	<b>149</b>
6.1	Hilbert Problem . . . . .	149
6.2	Cauchy Integral Equation of the First Kind . . . . .	153
6.3	Cauchy Integral Equation of the Second Kind . . . . .	157
6.4	Carleman Integral Equation . . . . .	161
6.5	Dispersion Relations . . . . .	166
6.6	Problems for Chapter 6 . . . . .	173
<b>7</b>	<b>Wiener–Hopf Method and Wiener–Hopf Integral Equation . . . . .</b>	<b>177</b>
7.1	The Wiener–Hopf Method for Partial Differential Equations . . . . .	177
7.2	Homogeneous Wiener–Hopf Integral Equation of the Second Kind . . . . .	191
7.3	General Decomposition Problem . . . . .	207
7.4	Inhomogeneous Wiener–Hopf Integral Equation of the Second Kind . . . . .	216
7.5	Toeplitz Matrix and Wiener–Hopf Sum Equation . . . . .	227
7.6	Wiener–Hopf Integral Equation of the First Kind and Dual Integral Equations . . . . .	235
7.7	Problems for Chapter 7 . . . . .	239
<b>8</b>	<b>Nonlinear Integral Equations . . . . .</b>	<b>249</b>
8.1	Nonlinear Integral Equation of Volterra type . . . . .	249
8.2	Nonlinear Integral Equation of Fredholm Type . . . . .	253
8.3	Nonlinear Integral Equation of Hammerstein type . . . . .	257
8.4	Problems for Chapter 8 . . . . .	259
<b>9</b>	<b>Calculus of Variations: Fundamentals . . . . .</b>	<b>263</b>
9.1	Historical Background . . . . .	263
9.2	Examples . . . . .	267
9.3	Euler Equation . . . . .	267
9.4	Generalization of the Basic Problems . . . . .	272
9.5	More Examples . . . . .	276
9.6	Differential Equations, Integral Equations, and Extremization of Integrals . . . . .	278
9.7	The Second Variation . . . . .	283

9.8	Weierstrass–Erdmann Corner Relation . . . . .	297
9.9	Problems for Chapter 9 . . . . .	300
<b>10</b>	<b>Calculus of Variations: Applications</b>	<b>303</b>
10.1	Feynman’s Action Principle in Quantum Mechanics . . . . .	303
10.2	Feynman’s Variational Principle in Quantum Statistical Mechanics . . . . .	308
10.3	Schwinger–Dyson Equation in Quantum Field Theory . . . . .	312
10.4	Schwinger–Dyson Equation in Quantum Statistical Mechanics . . . . .	329
10.5	Weyl’s Gauge Principle . . . . .	339
10.6	Problems for Chapter 10 . . . . .	356
	<b>Bibliography</b>	<b>365</b>
	<b>Index</b>	<b>373</b>