

1	Finite Mixture Modeling	1
1.1	Introduction	1
1.2	Finite Mixture Distributions	3
1.2.1	Basic Definitions	3
1.2.2	Some Descriptive Features of Finite Mixture Distributions	5
1.2.3	Diagnosing Similarity of Mixture Components	9
1.2.4	Moments of a Finite Mixture Distribution	10
1.2.5	Statistical Modeling Based on Finite Mixture Distributions	11
1.3	Identifiability of a Finite Mixture Distribution	14
1.3.1	Nonidentifiability Due to Invariance to Relabeling the Components	15
1.3.2	Nonidentifiability Due to Potential Overfitting	17
1.3.3	Formal Identifiability Constraints	19
1.3.4	Generic Identifiability	21
2	Statistical Inference for a Finite Mixture Model with Known Number of Components	25
2.1	Introduction	25
2.2	Classification for Known Component Parameters	26
2.2.1	Bayes' Rule for Classifying a Single Observation	26
2.2.2	The Bayes' Classifier for a Whole Data Set	27
2.3	Parameter Estimation for Known Allocation	29
2.3.1	The Complete-Data Likelihood Function	29
2.3.2	Complete-Data Maximum Likelihood Estimation	30
2.3.3	Complete-Data Bayesian Estimation of the Component Parameters	31
2.3.4	Complete-Data Bayesian Estimation of the Weights	35
2.4	Parameter Estimation When the Allocations Are Unknown	41
2.4.1	Method of Moments	42

2.4.2	The Mixture Likelihood Function	43
2.4.3	A Helicopter Tour of the Mixture Likelihood Surface for Two Examples	44
2.4.4	Maximum Likelihood Estimation	49
2.4.5	Bayesian Parameter Estimation	53
2.4.6	Distance-Based Methods	54
2.4.7	Comparing Various Estimation Methods	54
3	Practical Bayesian Inference for a Finite Mixture Model with Known Number of Components	57
3.1	Introduction	57
3.2	Choosing the Prior for the Parameters of a Mixture Model	58
3.2.1	Objective and Subjective Priors	58
3.2.2	Improper Priors May Cause Improper Mixture Posteriors	59
3.2.3	Conditionally Conjugate Priors	60
3.2.4	Hierarchical Priors and Partially Proper Priors	61
3.2.5	Other Priors	62
3.2.6	Invariant Prior Distributions	62
3.3	Some Properties of the Mixture Posterior Density	63
3.3.1	Invariance of the Posterior Distribution	63
3.3.2	Invariance of Seemingly Component- Specific Functionals	64
3.3.3	The Marginal Posterior Distribution of the Allocations	65
3.3.4	Invariance of the Posterior Distribution of the Allocations	67
3.4	Classification Without Parameter Estimation	68
3.4.1	Single-Move Gibbs Sampling	69
3.4.2	The Metropolis–Hastings Algorithm	72
3.5	Parameter Estimation Through Data Augmentation and MCMC	73
3.5.1	Treating Mixture Models as a Missing Data Problem	73
3.5.2	Data Augmentation and MCMC for a Mixture of Poisson Distributions	74
3.5.3	Data Augmentation and MCMC for General Mixtures	76
3.5.4	MCMC Sampling Under Improper Priors	78
3.5.5	Label Switching	78
3.5.6	Permutation MCMC Sampling	81
3.6	Other Monte Carlo Methods Useful for Mixture Models	83
3.6.1	A Metropolis–Hastings Algorithm for the Parameters	83
3.6.2	Importance Sampling for the Allocations	84
3.6.3	Perfect Sampling	85

3.7	Bayesian Inference for Finite Mixture Models Using Posterior Draws	85
3.7.1	Sampling Representations of the Mixture Posterior Density	85
3.7.2	Using Posterior Draws for Bayesian Inference	87
3.7.3	Predictive Density Estimation	89
3.7.4	Individual Parameter Inference	91
3.7.5	Inference on the Hyperparameter of a Hierarchical Prior	92
3.7.6	Inference on Component Parameters	92
3.7.7	Model Identification	94
4	Statistical Inference for Finite Mixture Models Under Model Specification Uncertainty	99
4.1	Introduction	99
4.2	Parameter Estimation Under Model Specification Uncertainty	100
4.2.1	Maximum Likelihood Estimation Under Model Specification Uncertainty	100
4.2.2	Practical Bayesian Parameter Estimation for Overfitting Finite Mixture Models	103
4.2.3	Potential Overfitting	105
4.3	Informal Methods for Identifying the Number of Components	107
4.3.1	Mode Hunting in the Mixture Posterior	108
4.3.2	Mode Hunting in the Sample Histogram	109
4.3.3	Diagnosing Mixtures Through the Method of Moments	110
4.3.4	Diagnosing Mixtures Through Predictive Methods	112
4.3.5	Further Approaches	114
4.4	Likelihood-Based Methods	114
4.4.1	The Likelihood Ratio Statistic	114
4.4.2	AIC, BIC, and the Schwarz Criterion	116
4.4.3	Further Approaches	117
4.5	Bayesian Inference Under Model Uncertainty	117
4.5.1	Trans-Dimensional Bayesian Inference	117
4.5.2	Marginal Likelihoods	118
4.5.3	Bayes Factors for Model Comparison	119
4.5.4	Formal Bayesian Model Selection	121
4.5.5	Choosing Priors for Model Selection	122
4.5.6	Further Approaches	123

5	Computational Tools for Bayesian Inference for Finite Mixtures Models Under Model Specification Uncertainty ..	125
5.1	Introduction	125
5.2	Trans-Dimensional Markov Chain Monte Carlo Methods	125
5.2.1	Product-Space MCMC	126
5.2.2	Reversible Jump MCMC	129
5.2.3	Birth and Death MCMC Methods	137
5.3	Marginal Likelihoods for Finite Mixture Models	139
5.3.1	Defining the Marginal Likelihood	139
5.3.2	Choosing Priors for Selecting the Number of Components	141
5.3.3	Computation of the Marginal Likelihood for Mixture Models	143
5.4	Simulation-Based Approximations of the Marginal Likelihood	143
5.4.1	Some Background on Monte Carlo Integration	143
5.4.2	Sampling-Based Approximations for Mixture Models ..	144
5.4.3	Importance Sampling	146
5.4.4	Reciprocal Importance Sampling	147
5.4.5	Harmonic Mean Estimator	148
5.4.6	Bridge Sampling Technique	150
5.4.7	Comparison of Different Simulation-Based Estimators	154
5.4.8	Dealing with Hierarchical Priors	159
5.5	Approximations to the Marginal Likelihood Based on Density Ratios	159
5.5.1	The Posterior Density Ratio	159
5.5.2	Chib's Estimator	160
5.5.3	Laplace Approximation	164
5.6	Reversible Jump MCMC Versus Marginal Likelihoods?	165
6	Finite Mixture Models with Normal Components	169
6.1	Finite Mixtures of Normal Distributions	169
6.1.1	Model Formulation	169
6.1.2	Parameter Estimation for Mixtures of Normals	171
6.1.3	The Kiefer-Wolfowitz Example	174
6.1.4	Applications of Mixture of Normal Distributions	176
6.2	Bayesian Estimation of Univariate Mixtures of Normals	177
6.2.1	Bayesian Inference When the Allocations Are Known	177
6.2.2	Standard Prior Distributions	179
6.2.3	The Influence of the Prior on the Variance Ratio	179
6.2.4	Bayesian Estimation Using MCMC	180
6.2.5	MCMC Estimation Under Standard Improper Priors ..	182

6.2.6	Introducing Prior Dependence Among the Components	185
6.2.7	Further Sampling-Based Approaches	187
6.2.8	Application to the Fishery Data	188
6.3	Bayesian Estimation of Multivariate Mixtures of Normals	190
6.3.1	Bayesian Inference When the Allocations Are Known	190
6.3.2	Prior Distributions	192
6.3.3	Bayesian Parameter Estimation Using MCMC	193
6.3.4	Application to Fisher's Iris Data	195
6.4	Further Issues	195
6.4.1	Parsimonious Finite Normal Mixtures	195
6.4.2	Model Selection Problems for Mixtures of Normals	199
7	Data Analysis Based on Finite Mixtures	203
7.1	Model-Based Clustering	203
7.1.1	Some Background on Cluster Analysis	203
7.1.2	Model-Based Clustering Using Finite Mixture Models	204
7.1.3	The Classification Likelihood and the Bayesian MAP Approach	207
7.1.4	Choosing Clustering Criteria and the Number of Components	210
7.1.5	Model Choice for the Fishery Data	216
7.1.6	Model Choice for Fisher's Iris Data	218
7.1.7	Bayesian Clustering Based on Loss Functions	220
7.1.8	Clustering for Fisher's Iris Data	224
7.2	Outlier Modeling	224
7.2.1	Outlier Modeling Using Finite Mixtures	224
7.2.2	Bayesian Inference for Outlier Models Based on Finite Mixtures	225
7.2.3	Outlier Modeling of Darwin's Data	226
7.2.4	Clustering Under Outliers and Noise	227
7.3	Robust Finite Mixtures Based on the Student- <i>t</i> Distribution	230
7.3.1	Parameter Estimation	230
7.3.2	Dealing with Unknown Number of Components	233
7.4	Further Issues	233
7.4.1	Clustering High-Dimensional Data	233
7.4.2	Discriminant Analysis	235
7.4.3	Combining Classified and Unclassified Observations	236
7.4.4	Density Estimation Using Finite Mixtures	237
7.4.5	Finite Mixtures as an Auxiliary Computational Tool in Bayesian Analysis	238

8	Finite Mixtures of Regression Models	241
8.1	Introduction	241
8.2	Finite Mixture of Multiple Regression Models	242
8.2.1	Model Definition	242
8.2.2	Identifiability	243
8.2.3	Statistical Modeling Based on Finite Mixture of Regression Models	246
8.2.4	Outliers in a Regression Model	249
8.3	Statistical Inference for Finite Mixtures of Multiple Regression Models	249
8.3.1	Maximum Likelihood Estimation	249
8.3.2	Bayesian Inference When the Allocations Are Known	250
8.3.3	Choosing Prior Distributions	252
8.3.4	Bayesian Inference When the Allocations Are Unknown	253
8.3.5	Bayesian Inference Using Posterior Draws	254
8.3.6	Dealing with Model Specification Uncertainty	255
8.4	Mixed-Effects Finite Mixtures of Regression Models	256
8.4.1	Model Definition	256
8.4.2	Choosing Priors for Bayesian Estimation	256
8.4.3	Bayesian Parameter Estimation When the Allocations Are Known	257
8.4.4	Bayesian Parameter Estimation When the Allocations Are Unknown	258
8.5	Finite Mixture Models for Repeated Measurements	259
8.5.1	Pooling Information Across Similar Units	260
8.5.2	Finite Mixtures of Random-Effects Models	260
8.5.3	Choosing the Prior for Bayesian Estimation	265
8.5.4	Bayesian Parameter Estimation When the Allocations Are Known	265
8.5.5	Practical Bayesian Estimation Using MCMC	267
8.5.6	Dealing with Model Specification Uncertainty	269
8.5.7	Application to the Marketing Data	270
8.6	Further Issues	273
8.6.1	Regression Modeling Based on Multivariate Mixtures of Normals	273
8.6.2	Modeling the Weight Distribution	274
8.6.3	Mixtures-of-Experts Models	274
9	Finite Mixture Models with Nonnormal Components	277
9.1	Finite Mixtures of Exponential Distributions	277
9.1.1	Model Formulation and Parameter Estimation	277
9.1.2	Bayesian Inference	278
9.2	Finite Mixtures of Poisson Distributions	279

9.2.1	Model Formulation and Estimation	279
9.2.2	Capturing Overdispersion in Count Data	280
9.2.3	Modeling Excess Zeros	282
9.2.4	Application to the Eye Tracking Data	283
9.3	Finite Mixture Models for Binary and Categorical Data	286
9.3.1	Finite Mixtures of Binomial Distributions	286
9.3.2	Finite Mixtures of Multinomial Distributions	288
9.4	Finite Mixtures of Generalized Linear Models	289
9.4.1	Finite Mixture Regression Models for Count Data	290
9.4.2	Finite Mixtures of Logit and Probit Regression Models	292
9.4.3	Parameter Estimation for Finite Mixtures of GLMs	293
9.4.4	Model Selection for Finite Mixtures of GLMs	294
9.5	Finite Mixture Models for Multivariate Binary and Categorical Data	294
9.5.1	The Basic Latent Class Model	295
9.5.2	Identification and Parameter Estimation	296
9.5.3	Extensions of the Basic Latent Class Model	297
9.6	Further Issues	298
9.6.1	Finite Mixture Modeling of Mixed-Mode Data	298
9.6.2	Finite Mixtures of GLMs with Random Effects	299
10	Finite Markov Mixture Modeling	301
10.1	Introduction	301
10.2	Finite Markov Mixture Distributions	301
10.2.1	Basic Definitions	302
10.2.2	Irreducible Aperiodic Markov Chains	304
10.2.3	Moments of a Markov Mixture Distribution	308
10.2.4	The Autocorrelation Function of a Process Generated by a Markov Mixture Distribution	310
10.2.5	The Autocorrelation Function of the Squared Process	311
10.2.6	The Standard Finite Mixture Distribution as a Limiting Case	312
10.2.7	Identifiability of a Finite Markov Mixture Distribution	313
10.3	Statistical Modeling Based on Finite Markov Mixture Distributions	314
10.3.1	The Basic Markov Switching Model	314
10.3.2	The Markov Switching Regression Model	315
10.3.3	Nonergodic Markov Chains	316
10.3.4	Relaxing the Assumptions of the Basic Markov Switching Model	316

11	Statistical Inference for Markov Switching Models	319
11.1	Introduction	319
11.2	State Estimation for Known Parameters	319
11.2.1	Statistical Inference About the States	320
11.2.2	Filtered State Probabilities	320
11.2.3	Filtering for Special Cases	323
11.2.4	Smoothing the States	324
11.2.5	Filtering and Smoothing for More General Models	326
11.3	Parameter Estimation for Known States	327
11.3.1	The Complete-Data Likelihood Function	327
11.3.2	Complete-Data Bayesian Parameter Estimation	329
11.3.3	Complete-Data Bayesian Estimation of the Transition Matrix	329
11.4	Parameter Estimation When the States are Unknown	330
11.4.1	The Markov Mixture Likelihood Function	330
11.4.2	Maximum Likelihood Estimation	333
11.4.3	Bayesian Estimation	334
11.4.4	Alternative Estimation Methods	334
11.5	Bayesian Parameter Estimation with Known Number of States	335
11.5.1	Choosing the Prior for the Parameters of a Markov Mixture Model	335
11.5.2	Some Properties of the Posterior Distribution of a Markov Switching Model	336
11.5.3	Parameter Estimation Through Data Augmentation and MCMC	337
11.5.4	Permutation MCMC Sampling	340
11.5.5	Sampling the Unknown Transition Matrix	340
11.5.6	Sampling Posterior Paths of the Hidden Markov Chain	342
11.5.7	Other Sampling-Based Approaches	345
11.5.8	Bayesian Inference Using Posterior Draws	345
11.6	Statistical Inference Under Model Specification Uncertainty	346
11.6.1	Diagnosing Markov Switching Models	346
11.6.2	Likelihood-Based Methods	346
11.6.3	Marginal Likelihoods for Markov Switching Models	347
11.6.4	Model Space MCMC	348
11.6.5	Further Issues	348
11.7	Modeling Overdispersion and Autocorrelation in Time Series of Count Data	348
11.7.1	Motivating Example	348
11.7.2	Capturing Overdispersion and Autocorrelation Using Poisson Markov Mixture Models	349
11.7.3	Application to the Lamb Data	351

12	Nonlinear Time Series Analysis Based on Markov	
	Switching Models	357
12.1	Introduction	357
12.2	The Markov Switching Autoregressive Model	358
12.2.1	Motivating Example	358
12.2.2	Model Definition	360
12.2.3	Features of the MSAR Model	362
12.2.4	Markov Switching Models for Nonstationary Time Series	363
12.2.5	Parameter Estimation and Model Selection	365
12.2.6	Application to Business Cycle Analysis of the U.S. GDP Data	365
12.3	Markov Switching Dynamic Regression Models	371
12.3.1	Model Definition	371
12.3.2	Bayesian Estimation	371
12.4	Prediction of Time Series Based on Markov Switching Models	372
12.4.1	Flexible Predictive Distributions	372
12.4.2	Forecasting of Markov Switching Models via Sampling-Based Methods	374
12.5	Markov Switching Conditional Heteroscedasticity	375
12.5.1	Motivating Example	375
12.5.2	Capturing Features of Financial Time Series Through Markov Switching Models	377
12.5.3	Switching ARCH Models	378
12.5.4	Statistical Inference for Switching ARCH Models	380
12.5.5	Switching GARCH Models	383
12.6	Some Extensions	384
12.6.1	Time-Varying Transition Matrices	384
12.6.2	Markov Switching Models for Longitudinal and Panel Data	385
12.6.3	Markov Switching Models for Multivariate Time Series	386
13	Switching State Space Models	389
13.1	State Space Modeling	389
13.1.1	The Local Level Model with and Without Switching	389
13.1.2	The Linear Gaussian State Space Form	391
13.1.3	Multiprocess Models	393
13.1.4	Switching Linear Gaussian State Space Models	393
13.1.5	The General State Space Form	394
13.2	Nonlinear Time Series Analysis Based on Switching State Space Models	396
13.2.1	ARMA Models with and Without Switching	396

13.2.2	Unobserved Component Time Series Models	397
13.2.3	Capturing Sudden Changes in Time Series	398
13.2.4	Switching Dynamic Factor Models	400
13.2.5	Switching State Space Models as a Semi-Parametric Smoothing Device	401
13.3	Filtering for Switching Linear Gaussian State Space Models	401
13.3.1	The Filtering Problem	402
13.3.2	Bayesian Inference for a General Linear Regression Model	402
13.3.3	Filtering for the Linear Gaussian State Space Model	404
13.3.4	Filtering for Multiprocess Models	406
13.3.5	Approximate Filtering for Switching Linear Gaussian State Space Models	406
13.4	Parameter Estimation for Switching State Space Models	410
13.4.1	The Likelihood Function of a State Space Model	411
13.4.2	Maximum Likelihood Estimation	412
13.4.3	Bayesian Inference	412
13.5	Practical Bayesian Estimation Using MCMC	415
13.5.1	Various Data Augmentation Schemes	416
13.5.2	Sampling the Continuous State Process from the Smoother Density	417
13.5.3	Sampling the Discrete States for a Switching State Space Model	420
13.6	Further Issues	421
13.6.1	Model Specification Uncertainty in Switching State Space Modeling	421
13.6.2	Auxiliary Mixture Sampling for Nonlinear and Nonnormal State Space Models	422
13.7	Illustrative Application to Modeling Exchange Rate Data	423

A	Appendix	431
A.1	Summary of Probability Distributions	431
A.1.1	The Beta Distribution	431
A.1.2	The Binomial Distribution	432
A.1.3	The Dirichlet Distribution	432
A.1.4	The Exponential Distribution	433
A.1.5	The F-Distribution	433
A.1.6	The Gamma Distribution	434
A.1.7	The Geometric Distribution	435
A.1.8	The Multinomial Distribution	435
A.1.9	The Negative Binomial Distribution	435
A.1.10	The Normal Distribution	436
A.1.11	The Poisson Distribution	437
A.1.12	The Student- <i>t</i> Distribution	437
A.1.13	The Uniform Distribution	438

A.1.14 The Wishart Distribution	438
A.2 Software	439
References	441
Index	481