Contents

Preface to the English Edition ix

Preface to the German Edition xi

Chapter 1. Notations and Prerequisites from Analysis 1

Chapter 2. Curves in $\mathbb{R}^n$ 7

2A Frenet curves in $\mathbb{R}^n$ 7

2B Plane curves and space curves 14

2C Relations between the curvature and the torsion 20

2D The Frenet equations and the fundamental theorem of the local theory of curves 27

2E Curves in Minkowski space $\mathbb{R}^3_1$ 33

2F The global theory of curves 37

Exercises 50

Chapter 3. The Local Theory of Surfaces 55

3A Surface elements and the first fundamental form 56

3B The Gauss map and the curvature of surfaces 66

3C Surfaces of rotation and ruled surfaces 77

3D Minimal surfaces 96

3E Surfaces in Minkowski space $\mathbb{R}^3_1$ 113

Bibliografische Informationen
http://d-nb.info/1106139399

digitalisiert durch DEUTSCHE NATIONAL BIBLIOTHEK

3F Hypersurfaces in $\mathbb{R}^{n+1}$ 122
Exercises 125

Chapter 4. The Intrinsic Geometry of Surfaces 133
4A The covariant derivative 134
4B Parallel displacement and geodesics 140
4C The Gauss equation and the Theorema Egregium 145
4D The fundamental theorem of the local theory of surfaces 152
4E The Gaussian curvature in special parameters 157
4F The Gauss-Bonnet Theorem 165
4G Selected topics in the global theory of surfaces 180
Exercises 192

Chapter 5. Riemannian Manifolds 197
5A The notion of a manifold 198
5B The tangent space 205
5C Riemannian metrics 212
5D The Riemannian connection 218

Chapter 6. The Curvature Tensor 233
6A Tensors 233
6B The sectional curvature 242
6C The Ricci tensor and the Einstein tensor 250

Chapter 7. Spaces of Constant Curvature 265
7A Hyperbolic space 266
7B Geodesics and Jacobi fields 276
7C The space form problem 291
7D Three-dimensional Euclidean and spherical space forms 296
Exercises 306

Chapter 8. Einstein Spaces 309
8A The variation of the Hilbert-Einstein functional 312
8B The Einstein field equations 321

8C Homogenous Einstein spaces 325
8D The decomposition of the curvature tensor 331
8E The Weyl tensor 341
8F Duality for four-manifolds and Petrov types 350
Exercises 358

Solutions to selected exercises 361

Bibliography 391

List of notation 395

Index 397