Contents

	Prefac A Brie	e if Review of Linear Algebra	xi 1
	Part	I: Groups: Discrete or Continuous, Finite or Infinite	
	I.1	Symmetry and Groups	37
	1.2	Finite Groups	55
	1.3	Rotations and the Notion of Lie Algebra	70
		II: Representing Group Elements by Matrices	
	II.1	Representation Theory	89
	II.2	Schur's Lemma and the Great Orthogonality Theorem	101
	II.3	Character Is a Function of Class	114
	II.4	Real, Pseudoreal, Complex Representations, and the Number	
		of Square Roots	136
	II.i1	Crystals Are Beautiful	146
	II.i2	Euler's φ -Function, Fermat's Little Theorem, and Wilson's Theorem	150

III Part III: Group Theory in a Quantum World

III.1	Quantum Mechanics and Group Theory: Parity, Bloch's Theorem,	
	and the Brillouin Zone	161
III.2	Group Theory and Harmonic Motion: Zero Modes	168
III.3	Symmetry in the Laws of Physics: Lagrangian and Hamiltonian	176

viii | Contents

IV Part IV: Tensor, Covering, and Manifold

IV.1	Tensors and Representations of the Rotation Groups $SO(N)$	185
IV.2	Lie Algebra of SO(3) and Ladder Operators: Creation and Annihilation	203
IV.3	Angular Momentum and Clebsch-Gordan Decomposition	216
IV.4	Tensors and Representations of the Special Unitary Groups $SU(N)$	227
IV.5	SU(2): Double Covering and the Spinor	244
IV.6	The Electron Spin and Kramer's Degeneracy	255
IV.7	Integration over Continuous Groups, Topology, Coset Manifold,	
	and SO(4)	261
IV.8	Symplectic Groups and Their Algebras	277
IV.9	From the Lagrangian to Quantum Field Theory: It Is but a Skip	
	and a Hop	283
IV.i1	Multiplying Irreducible Representations of Finite Groups:	
	Return to the Tetrahedral Group	289
IV.i2	Crystal Field Splitting	292
IV.i3	Group Theory and Special Functions	295
IV.i4	Covering the Tetrahedron	299

V Part V: Group Theory in the Microscopic World

V.1	Isospin and the Discovery of a Vast Internal Space	303
V.2	The Eightfold Way of <i>SU</i> (3)	312
V.3	The Lie Algebra of $SU(3)$ and Its Root Vectors	325
V.4	Group Theory Guides Us into the Microscopic World	337

VI Part VI: Roots, Weights, and Classification of Lie Algebras

VI.1	The Poor Man Finds His Roots	347
VI.2	Roots and Weights for Orthogonal, Unitary, and Symplectic Algebras	350
VI.3	Lie Algebras in General	364
VI.4	The Killing-Cartan Classification of Lie Algebras	376
VI.5	Dynkin Diagrams	384

VII Part VII: From Galileo to Majorana

VII.1	Spinor Representations of Orthogonal Algebras	405
VII.2	The Lorentz Group and Relativistic Physics	428
VII.3	SL(2,C) Double Covers SO(3,1): Group Theory Leads Us	
	to the Weyl Equation	450
VII.4	From the Weyl Equation to the Dirac Equation	468
VII.5	Dirac and Majorana Spinors: Antimatter and Pseudoreality	482
VII.i1	A Hidden SO(4) Algebra in the Hydrogen Atom	491

Contents | ix

	VII.i2 The Unexpected Emergence of the Dirac Equation in Condensed Matter PhysicsVII.i3 The Even More Unexpected Emergence of the Majorana Equation in Condensed Matter Physics	497 501
VIII	Part VIII: The Expanding Universe	
	VIII.1 Contraction and Extension	507
	VIII.2 The Conformal Algebra	515
	VIII.3 The Expanding Universe from Group Theory	523
IX	Part IX: The Gauged UniverseIX.1The Gauged UniverseIX.2Grand Unification and SU(5)IX.3From SU(5) to SO(10)IX.4The Family Mystery	531 541 550 560
	Epilogue	565
	Timeline of Some of the People Mentioned	567
	Solutions to Selected Exercises	569
	Bibliography	581
	Index	583
	Collection of Formulas	601