

Contents

1	Introduction	1
1.1	The Riemann Integral	1
1.2	An Example of Riemann Integration	5
1.3	The Lebesgue Integral	7
1.4	'Interesting' and 'Inside'	10
1.5	Some Examples of Tricks	12
1.6	Singularities	17
1.7	Dalzell's Integral	22
1.8	Where Integrals Come From	24
1.9	Last Words	39
1.10	Challenge Problems	40
2	'Easy' Integrals	43
2.1	Six 'Easy' Warm-Ups	43
2.2	A New Trick	48
2.3	Two Old Tricks, Plus a New One	55
2.4	Another Old Trick: Euler's Log-Sine Integral	64
2.5	Challenge Problems	70
3	Feynman's Favorite Trick	73
3.1	Leibniz's Formula	73
3.2	An Amazing Integral	83
3.3	Frullani's Integral	84
3.4	The Flip-Side of Feynman's Trick	88
3.5	Combining Two Tricks	98
3.6	Uhler's Integral and Symbolic Integration	102
3.7	The Probability Integral Revisited	105
3.8	Dini's Integral	109
3.9	Feynman's Favorite Trick Solves a Physics Equation	112
3.10	Challenge Problems	114

4	Gamma and Beta Function Integrals	117
4.1	Euler's Gamma Function	117
4.2	Wallis' Integral and the Beta Function	119
4.3	Double Integration Reversal	130
4.4	The Gamma Function Meets Physics	141
4.5	Challenge Problems	145
5	Using Power Series to Evaluate Integrals	149
5.1	Catalan's Constant	149
5.2	Power Series for the Log Function	153
5.3	Zeta Function Integrals	161
5.4	Euler's Constant and Related Integrals	167
5.5	Challenge Problems	183
6	Seven Not-So-Easy Integrals	187
6.1	Bernoulli's Integral	187
6.2	Ahmed's Integral	190
6.3	Coxeter's Integral	194
6.4	The Hardy-Schuster Optical Integral	201
6.5	The Watson/van Peype Triple Integrals	206
6.6	Elliptic Integrals in a Physical Problem	212
6.7	Challenge Problems	219
7	Using $\sqrt{-1}$ to Evaluate Integrals	225
7.1	Euler's Formula	225
7.2	The Fresnel Integrals	227
7.3	$\zeta(3)$ and More Log-Sine Integrals	231
7.4	$\zeta(2)$, At Last!	236
7.5	The Probability Integral <i>Again</i>	240
7.6	Beyond Dirichlet's Integral	241
7.7	Dirichlet Meets the Gamma Function	249
7.8	Fourier Transforms and Energy Integrals	252
7.9	'Weird' Integrals from Radio Engineering	257
7.10	Causality and Hilbert Transform Integrals	267
7.11	Challenge Problems	275
8	Contour Integration	279
8.1	Prelude	279
8.2	Line Integrals	280
8.3	Functions of a Complex Variable	282
8.4	The Cauchy-Riemann Equations and Analytic Functions	289
8.5	Green's Integral Theorem	292
8.6	Cauchy's First Integral Theorem	295
8.7	Cauchy's Second Integral Theorem	309
8.8	Singularities and the Residue Theorem	323
8.9	Integrals with Multi-valued Integrand	331
8.10	Challenge Problems	339

9	Epilogue	343
9.1	Riemann, Prime Numbers, and the Zeta Function	343
9.2	Deriving the Functional Equation for $\zeta(s)$	352
9.3	Challenge Questions	365
	Solutions to the Challenge Problems	369
	Index	409