

Contents

<i>Preface</i>	vii
<i>Introduction</i>	ix
<i>About the Authors</i>	xv
<i>Acknowledgments</i>	xvii
<i>References to Front Matter</i>	xix
Chapter 1. The Golden Ratio, Fibonacci Numbers, and the “Golden” Hyperbolic Fibonacci and Lucas Functions	1
1.1 The Idea of Harmony and the Golden Section in the History of Science	1
1.2 Proclus’ Hypothesis: A New View on Euclid’s <i>Elements</i> and the History of Mathematics	3
1.3 The Golden Ratio in Euclid’s <i>Elements</i>	10
1.4 The Algebraic Identities for the Golden Ratio	16
1.5 Fibonacci Numbers	19
1.6 Lucas Numbers	25
1.7 Binet’s Formulas	27
1.8 The Theory of Fibonacci Numbers in Modern Mathematics	30
1.9 The “Golden” Hyperbolic Fibonacci and Lucas Functions	33
1.10 Hyperbolic Geometry of Phyllotaxis (Bodnar’s Geometry)	45
References	49

Chapter 2. The Mathematics of Harmony and General Theory of Recursive Hyperbolic Functions	51
2.1 The Mathematics of Harmony: The History, Generalizations and Applications of Fibonacci Numbers and Golden Ratio	51
2.2 Algorithmic Measurement Theory as a Constructive Measurement Theory Based on the Abstraction of Potential Infinity	56
2.3 Pascal's Triangle, Fibonacci p -Numbers, and Golden p -Proportions	57
2.4 Fibonacci p -codes	59
2.5 Codes of the Golden Proportion	59
2.6 The Golden Number Theory	60
2.7 Lucas Sequences	61
2.8 The Fibonacci λ -numbers	63
2.9 The Metallic Proportions	67
2.10 Gazale's Formulas	70
2.11 Hyperbolic Fibonacci and Lucas λ -functions	75
2.12 The Partial Cases of the λ -Fibonacci and λ -Lucas Hyperbolic Functions	77
2.13 The Most Important Formulas for the λ -Fibonacci and λ -Lucas Hyperbolic Functions	79
2.14 A General Theory of the Recursive Hyperbolic Functions	83
2.15 Conclusions for Chapter 2	85
References	86
 Chapter 3. Hyperbolic and Spherical Solutions of Hilbert's Fourth Problem: The Way to the Recursive Non-Euclidean Geometries	 89
3.1 Non-Euclidean Geometry	89
3.2 Hilbert's Problems and Hilbert's Philosophy	95
3.3 Klein's Icosahedral Idea	97
3.4 Hilbert's Fourth Problem	99
3.5 Hyperbolic Solution of Hilbert's Fourth Problem	102
3.6 Spherical Fibonacci Functions	122
3.7 Spherical Solution to Hilbert's Fourth Problem	132

3.8	Comparative Table for Hyperbolic and Spherical Solutions of Hilbert’s Fourth Problem	132
3.9	Searching for New Recursive Hyperbolic and Spherical Worlds of Nature: A New Challenge for the Theoretical Natural Sciences	133
3.10	Hilbert’s Fourth Problem as a Possible Candidate for the Millennium Problem in Geometry	143
	References	145

Chapter 4. Introduction to the “Golden” Qualitative Theory of Dynamical Systems Based on the Mathematics of Harmony 149

4.1	Beauty and Aesthetics of Mathematics	149
4.2	Preliminaries	154
4.3	Metallic Irrational Foliations without Singularities on the Two-dimensional Torus T^2	162
4.4	The Metallic Irrational Foliations with Four Needle Type Singularities on the Two-dimensional Sphere S^2	173
4.5	Anosov’s Automorphisms (Hyperbolic Automorphisms) on the Two-dimensional Torus T^2 and the Metallic Proportions	179
4.6	Prospects for Further Development of the Qualitative Theory of Dynamical Systems, Based on the Mathematics of Harmony	190
	References	199

Chapter 5. The Basic Stages of the Mathematical Solution to the Fine-Structure Constant Problem as a Physical Millennium Problem 207

5.1	Physical Millennium Problems	207
5.2	Classical Special Theory of Relativity	209
5.3	Fibonacci Special Theory of Relativity	212
5.4	The Fine-Structure Constant α and Its Relationship with the Evolution of the Universe	222

5.5	Quantitative Results of the Fibonacci Special Theory of Relativity from the Onset of the Big Bang $T = 0$ to any Time T [Billion Years]	244
5.6	Advantages of the Fibonacci Special Theory of Relativity in Comparison with the Classical Special Theory of Relativity	245
5.7	The Ratio of the Proton Mass M to the Electron Mass m Depending on the Universe Evolution	247
5.8	General Conclusions	249
	References	256
Appendix: From the “Golden” Geometry to the Multiverse		261
A.1	Conception of Multiverse	261
A.2	The Conceptions and Theories used in this Study	262
A.3	Mathematical Models of the Multiverse	265
A.4	Fundamental Physical Constants of the λ -Universes	268
A.5	The Mathematics of Harmony as an Essential Part of Mathematical Physics	271
	References	275
	<i>Index</i>	279