

# CONTENTS

List of Figures	xxi
<b>1 THE BIRTH OF A PARADIGM</b>	<b>1</b>
1.1 From the Great Wave to the Great War	1
1.1.1 Hydrodynamics	1
1.1.2 Nonlinear diffusion	3
1.1.3 Bäcklund transformation theory	6
1.1.4 A theory of matter	7
1.2 Between the wars	8
1.3 Nonlinear research from 1945 to 1985	11
1.3.1 Nerve studies	11
1.3.2 Autocatalytic chemical reactions	12
1.3.3 Solitons	14
1.3.4 Local modes in molecules and molecular crystals	19
1.3.5 Elementary particle research	20
1.4 Recent developments	21
References	23
<b>2 LINEAR WAVE THEORY</b>	<b>28</b>
2.1 Dispersionless linear equations	28
2.2 Dispersive linear equations	30
2.3 <i>The linear diffusion equation</i>	31
2.4 Driven systems	33
2.4.1 Green's method	33
2.4.2 Fredholm's theorem	35
2.5 Stability	37
2.5.1 General definitions	37
2.5.2 Linear stability	38
2.5.3 Signaling problems	39
2.6 Scattering theory	40
2.6.1 Solutions of Schrödinger's equation	40
2.6.2 Gel'fand-Levitan theory	43
2.6.3 A reflectionless potential	48
2.7 Problems	48
References	53

<b>3 THE CLASSICAL SOLITON EQUATIONS</b>	55
3.1 The Korteweg–de Vries (KdV) equation	57
3.1.1 Long water waves	57
3.1.2 Solitary wave solutions	58
3.1.3 Periodic solutions	59
3.1.4 A Bäcklund transformation for KdV	61
3.1.5 $N$ -soliton formulas	67
3.2 The sine–Gordon (SG) equation	71
3.2.1 Long Josephson junctions	71
3.2.2 Solitary waves	72
3.2.3 Periodic waves	74
3.2.4 Nonlinear standing waves	77
3.2.5 Two-soliton solutions	81
3.2.6 More spatial dimensions	85
3.3 The nonlinear Schrödinger (NLS) equation	88
3.3.1 Nonlinear wave packets	88
3.3.2 Modulated traveling-wave solutions of NLS(+)	90
3.3.3 Dark soliton solutions of NLS(-)	92
3.3.4 A BT for NLS(+)	93
3.3.5 Transverse phenomena	95
3.4 Summary	98
3.5 Problems	98
References	106
<b>4 REACTION-DIFFUSION SYSTEMS</b>	110
4.1 Simple reaction-diffusion equations	111
4.1.1 The Zeldovich–Frank–Kamenetsky (Z–F) equation	111
4.1.2 The Burgers equation	116
4.2 The Hodgkin–Huxley (H–H) system	117
4.2.1 Space-clamped squid membrane dynamics	118
4.2.2 The H–H impulse	124
4.3 Simplified nerve models	127
4.3.1 The Markin–Chizmadzhev (M–C) model	127
4.3.2 The FitzHugh–Nagumo (F–N) model	129
4.3.3 Morris–Lecar (M–L) models	134
4.4 Stability analyses	138
4.4.1 The Z–F equation	138
4.4.2 The M–C model	139
4.4.3 The F–N model	140
4.4.4 The H–H and M–L systems	143
4.5 Decremental conduction	143
4.6 Nonuniform fibers	147
4.6.1 Tapered fibers	147
4.6.2 Leading-edge charge and impulse ignition	149

4.6.3	Dendritic logic	150
4.7	More space dimensions	154
4.7.1	Two-dimensional nonlinear diffusion	154
4.7.2	Nonlinear diffusion in three dimensions	156
4.7.3	Turing patterns	159
4.7.4	Hypercycles	160
4.8	Summary	161
4.9	Problems	162
	References	171
<b>5</b>	<b>NONLINEAR LATTICES</b>	<b>176</b>
5.1	Spring-mass lattices	177
5.1.1	The Toda-lattice soliton	178
5.1.2	Lattice solitary waves	179
5.1.3	Existence of lattice solitary waves	180
5.1.4	Intrinsic localized modes and intrinsic gap modes	182
5.2	Lattices with nonlinear on-site potentials	185
5.2.1	The discrete sine-Gordon equation	187
5.2.2	Nonlinear Schrödinger lattices	190
5.2.3	The discrete self-trapping equation	197
5.3	Biological solitons	202
5.3.1	Alpha-helix solitons in protein	202
5.3.2	Self-trapping in globular proteins	205
5.3.3	Solitons in DNA	207
5.4	Nonconservative lattices	210
5.4.1	Quasiharmonic lattices	210
5.4.2	Myelinated nerves	215
5.4.3	Emergence of form by replication	219
5.5	Assemblies of neurons	221
5.6	Summary	223
5.7	Problems	224
	References	230
<b>6</b>	<b>INVERSE SCATTERING METHODS</b>	<b>238</b>
6.1	Linear scattering revisited	240
6.1.1	Scattering solutions, bound states, and upper half plane poles	240
6.1.2	Why the upper half plane poles must be simple	242
6.1.3	The Gel'fand-Levitan equation again	245
6.1.4	Any questions?	249
6.2	Inverse scattering method for KdV	250
6.2.1	General description	250
6.2.2	Some examples	252

6.2.3	Reduction to Fourier analysis in the small amplitude limit	257
6.3	Two-component scattering theory	258
6.3.1	Linear theory	258
6.3.2	ISMs for two-component scattering	264
6.4	The sine-Gordon equation	266
6.5	The nonlinear Schrödinger equation	271
6.6	Conservation laws	273
6.6.1	Conservation laws for the KdV equation	274
6.6.2	Conserved densities for matrix scattering	276
6.7	Summary	277
6.8	Problems	278
	References	285
<b>7</b>	<b>PERTURBATION THEORY</b>	287
7.1	Perturbed matrices	288
7.2	A damped harmonic oscillator	290
7.2.1	Energy analysis	290
7.2.2	Multiple time scales	291
7.3	Energy analysis of soliton dynamics	293
7.3.1	Korteweg-de Vries solitons	294
7.3.2	Sine-Gordon solitons	296
7.3.3	Nonlinear Schrödinger solitons	299
7.4	More general soliton analyses	301
7.4.1	Multiple scale analysis of an SG kink	301
7.4.2	Variational analysis of an NLS soliton	306
7.5	Multisoliton perturbation theory	309
7.5.1	General theory	310
7.5.2	Kink-antikink collisions	314
7.5.3	Radiation from a fluxon	317
7.6	Neural perturbations	319
7.6.1	The FitzHugh-Nagumo system	320
7.6.2	Electrodynamic (ephaptic) coupling of nerves	322
7.7	Summary	326
7.8	Problems	327
	References	335
<b>8</b>	<b>QUANTUM LATTICE SOLITONS</b>	337
8.1	Quantum oscillators	337
8.1.1	A classical nonlinear oscillator	337
8.1.2	The birth of quantum theory	339
8.1.3	A quantum linear oscillator	342
8.1.4	The rotating wave approximation	345
8.1.5	The Born-Oppenheimer approximation	347

8.1.6	Dirac's notation	350
8.1.7	Pump-probe measurements	351
8.2	Self-trapping in the dihalomethanes	353
8.2.1	Classical analysis	354
8.2.2	Quantum analysis	356
8.2.3	Comparison with experiments	360
8.3	Boson lattices	361
8.3.1	The discrete self-trapping equation	361
8.3.2	A lattice nonlinear Schrödinger equation	365
8.3.3	Soliton wave packets	370
8.3.4	The Hartree approximation	372
8.4	More general quanta	377
8.4.1	The Ablowitz-Ladik equation	377
8.4.2	Salerno's equation	380
8.4.3	A fermionic polaron model	381
8.4.4	The Hubbard model	384
8.5	Energy transport in protein	386
8.5.1	Dynamic equations	386
8.5.2	Experimental observations	390
8.5.3	Recent comments	398
8.6	A quantum lattice sine-Gordon equation	401
8.7	Theoretical perspectives	403
8.7.1	Number state method	403
8.7.2	Quantum inverse scattering method	404
8.7.3	QISM analysis of the DST dimer	406
8.7.4	Comparison of the NSM and the QISM	407
8.8	Summary	409
8.9	Problems	409
	References	420
<b>9</b>	<b>LOOKING AHEAD</b>	<b>424</b>
	References	431
<b>APPENDIX A</b>	<b>CONSERVATION LAWS AND CONSERVATIVE SYSTEMS</b>	<b>433</b>
	References	437
<b>APPENDIX B</b>	<b>MULTISOLITON FORMULAS</b>	<b>438</b>
B.1	The KdV equation	438
B.2	The SG equation	438
B.3	The NLS equation	440
B.4	The Toda lattice	440
	References	441

<b>APPENDIX C</b>	<b>ELLIPTIC FUNCTIONS</b>	443
References		447
<b>APPENDIX D</b>	<b>STABILITY OF NERVE IMPULSES</b>	448
References		454
<b>APPENDIX E</b>	<b>PERIODIC TODA-LATTICE SOLITONS</b>	456
References		457
<b>APPENDIX F</b>	<b>ANALYTIC APPROXIMATIONS FOR LONG LATTICE SOLITARY WAVES</b>	458
Reference		459
<b>APPENDIX G</b>	<b>MULTIPLE-SCALE ANALYSIS OF A DAMPED-HARMONIC OSCILLATOR</b>	460
References		462
<b>APPENDIX H</b>	<b>GREEN FUNCTIONS FOR SOLITON RADIATION</b>	463
References		467
<b>INDEX</b>		469