

Contents

Introduction	xi
I Fundamentals of Statistical Physics	1
1 The Lectures — A Survey	3
1.1 The Journey: Many Different Approaches	3
1.2 The Main Sights	5
1.3 Is the Trip Worthwhile?	9
2 One Particle and Many	11
2.1 Formulation	11
2.2 The Ising Model	12
2.3 N Independent Particles — Quantum Description	13
2.4 Averages From Derivatives	15
2.5 N Independent Particles in a Box	17
2.6 Fluctuations Big and Small	20
2.7 The Problems of Statistical Physics	21
3 Gaussian Distributions	29
3.1 Introduction	29
3.2 One Variable	29
3.3 Many Gaussian Variables	31
3.4 Lattice Green Function	33
3.5 Gaussian Random Functions	35
3.6 Central Limit Theorem	35
3.7 Distribution of Energies	36
3.8 Large Deviations	38
3.9 On Almost Gaussian Integrals	41
3.10 Three Versions of Gaussian Problems	42

4	Quantum Mechanics and Lattices	45
4.1	All of Quantum Mechanics in One Brief Section	45
4.2	From $d = 1$ Models to Quantum Mechanics	46
4.3	An Example: The Linear Ising Chain	48
4.4	One-Dimensional Gaussian Model	51
4.5	Coherence Length	56
4.6	Operator Averages	57
4.7	Correlation Functions	59
4.8	Ising Correlations	60
4.9	Two-Dimensional Ising Model	64
II	Random Dynamics	69
5	Diffusion and Hopping	71
5.1	Random Walk on a Lattice	71
5.2	Formulating This Problem	73
5.3	The Diffusion of Probability and Particles	76
5.4	From Conservation to Hydrodynamic Equations	79
5.5	Distribution Functions	83
5.6	Cascade Processes and Securities Prices	84
5.7	Reprints on Dynamics	91
5.7.1	Forest and Witten: Smoke Particle Aggregates	92
5.7.2	Witten and Sander: Diffusion Limited Aggregation	101
5.7.3	Kadanoff: Chaos and Complexity	105
6	From Hops to Statistical Mechanics	119
6.1	Random Walk in Momentum	120
6.2	The Diffusion Equation Again	123
6.3	Time Dependence of Probability	124
6.4	Time Dependence in Deterministic Case	126
6.5	Equilibrium Solutions	128
6.6	Back to Collisions	131
6.7	From Fokker-Planck to Equilibrium	133
6.8	Properties of Fokker-Planck Equation	135
6.9	Reprints on Organization	138
6.9.1	Chao Tang <i>et al.</i> : Phase Organization	139
6.9.2	Bak <i>et al.</i> : Self-Organized Criticality	143
6.9.3	Carlson <i>et al.</i> : Singular Diffusion	147
6.9.4	Jaeger <i>et al.</i> : Experimental Studies	151

7	Correlations and Response	155
7.1	Time Independent Response	155
7.2	Hamiltonian Time-Dependence	158
7.3	Sum Rules	161
7.4	Non-Interacting Particles	163
7.5	Plasma Behavior	164
 III More Statistical Mechanics		 169
8	Statistical Thermodynamics	171
8.1	The Chemical Potential Defined	171
8.2	Barometer Formula	173
8.3	Sharing Energy	174
8.4	Ensemble Theory	179
8.5	Temperatures and Energy Flow	182
9	Fermi, Bose, and Other	187
9.1	Quantum Formulation	187
9.2	Statistical Mechanics of Non-Interacting Degenerate Particles	188
9.3	The Non-Degenerate Limit	191
9.4	Degenerate Fermions	192
9.5	Degenerate Bosons I. Photons and Phonons	196
9.6	Degenerate Bosons II. One-Dimensional Phonons	198
9.7	Degenerate Bosons III. Bose Phase Transition	201
9.8	Entropies	203
 IV Phase Transitions		 207
10	Overview of Phase Transitions	209
10.1	Thermodynamic Phases	209
10.2	Phase Transitions	210
10.3	Two Kinds of Transitions	211
10.4	Back to the Ising Model	214
10.5	Mean Field Theory of Magnets	215
10.6	The Phases	216
10.7	Low Temperature Result	218
10.8	Free Energy Selection Argument	219
10.9	Behaviors of Different Phases	221
11	Mean Field Theory of Critical Behavior	225
11.1	The Infinite Range Model	226
11.2	Mean Field Theory Near the Critical Point	227

11.3	Critical Indices	230
11.4	Scaling Function for Magnetization	231
11.5	Spatial Correlations	232
11.6	Analyticity	238
11.7	Mean Field Theory for the Free Energy	239
11.8	When Mean Field Theory Fails	242
12	Continuous Phase Transitions	247
12.1	Historical Background	247
12.2	Widom Scaling Theory	248
12.3	The Ising Model: Rescaled	252
12.4	Fixed Points	257
12.5	Phenomenology of Scaling Fields	258
12.6	Theory of Scaling Fields	259
12.7	Scaling Relations for Operators	262
12.8	Transforming Operators	266
12.9	Universality	266
12.10	Operator Product Expansions	267
12.11	Reprints on Critical Correlations	268
	12.11.1 Kadanoff: Correlations Along a Line	269
	12.11.2 Kadanoff–Wegner: Marginal Behavior	274
13	Renormalization in One Dimension	279
13.1	Introduction	279
13.2	Decimation	279
13.3	The Ising Example	280
13.4	Phase Diagrams, Flow Diagrams, and the Coherence Length	281
13.5	The Gaussian Model	283
13.6	Analysis of Recursion Relation	284
13.7	Fixed Point Analysis for the Gaussian Model	285
13.8	Two-Dimensional Ising Model	288
14	Real Space Renormalization Techniques	291
14.1	Introduction	291
14.2	Decimation: An Exact Calculation	292
14.3	The Method of Neglect	294
14.4	Potential Moving	295
14.5	Further Work	298
14.6	Reprints on Real Space RG	298
	14.6.1 Niemeijer and van Leeuwen: Triangular Lattice R.G.	299
	14.6.2 David Nelson’s Early Summary	303
	14.6.3 Kadanoff: Bond-moving, and a Variational Method	308
	14.6.4 Kadanoff: Migdal’s Simple and Versatile Method	312
	14.6.5 Migdal’s Original Papers	348

15 Duality	359
15.1 Doing Sums	359
15.2 Two Dimensions	361
15.3 Direct Coupling and Dual Coupling	363
15.4 Two-Dimensional Calculation	365
15.5 Ising Model	368
15.6 XY is Connected to SOS	369
15.7 Gaussian goes into Gaussian	371
15.8 Dual Correlations	371
16 Planar Model and Coulomb Systems	377
16.1 Why Study a Planar Model?	377
16.2 One-Dimensional Case	379
16.3 Phases of the Planar Model	380
16.4 The Gaussian Approximation	382
16.5 Two-Dimensional Coulomb Systems	386
16.6 Multipole Expansion	387
16.7 Reprint on Spin Waves	390
16.7.1 V. L. Berezinskii: An Overview of Problems with Continuous Symmetry	391
17 XY Model, Renormalization, and Duality	399
17.1 Plan of Action	399
17.2 Villain Representation of the Basic Bonds	400
17.3 Duality Transformation	401
17.4 Two Limits	402
17.5 Vortex Representation	403
17.6 The Magnetically Charged System	405
17.7 Correlation Calculation	408
17.8 The Renormalization Calculation	409
17.9 Spatial Averages	411
17.10 The Actual Renormalization	413
17.11 Reprints on Planar Model	415
17.11.1 The Kosterlitz–Thouless Theory	416
17.11.2 Kosterlitz: On Renormalization of the Planar Model	439
17.11.3 Jorge V. José, Leo P. Kadanoff, Scott Kirkpatrick, David R. Nelson: Renormalization and Vortices	454
Index	479