Contents

1.	Int	roduct	ion	1	
2.	Vortex-Dominated Flows and General Theory			7	
	2.1	Gover	ning Equations for Compressible Viscous Vortical Flows	9	
	2.2	The C	Consistency Conditions and Time Invariants	13	
		2.2.1	Derivation of the Consistency Conditions	14	
		2.2.2	Derivation of the Time Invariants	16	
		2.2.3	Axisymmetric Flows	18	
		2.2.4	Two-Dimensional Flows	19	
	2.3	Incom	pressible Vortical Flows	23	
		2.3.1	Far-Field Vector Velocity Potential	26	
		2.3.2	Reduction of the Vector Velocity Potential		
			in the Far Field to the Corresponding Scalar Potential	28	
		2.3.3	Relating the Coefficients in the Scalar Potentials		
			of <i>n</i> th Order Directly to <i>n</i> th Moments of Vorticity	32	
3.	Motion and Decay				
	of Vortex Filaments – Matched Asymptotics			35	
	3.1	Invisc	id Theories of Vortical Flows		
		and T	heir Deficiencies	40	
		3.1.1	Potential Flows Around a Spinning Disc		
			and a Rankine Vortex	43	
		3.1.2	Potential Flow Induced by a Vortex Line	53	
	3.2	Plana	r Motion of Vortex Spots		
		and T	'heir Core Structures	58	
		3.2.1	The Inner Solution	62	
		3.2.2	Leading-Order Core Structure	69	
		3.2.3	Asymmetric First Order Solution in the Normal Time		
			Scale and the Velocity of the Vortex	72	
		3.2.4	Asymmetric First Order Two-Time Solution		
			and the Oscillatory Motion of the Vortex Center	74	
	3.3	Motio	n of Slender Vortex Filaments and Evolution		
		of The	eir Core Structures	81	
		3.3.1	The Expansion Scheme	87	

				00
		3.3.2	The Leading-Order Equations	89
		3.3.3	The First Order Equations and the Asymmetric	01
			Solutions	91
		3.3.4	Alternative Derivation: Vorticity-Based Analysis	90
		3.3.5	Asymptotic Solution of the Vorticity Transport	106
			Equation	100
		3.3.6	The Second-Order Equations and the Evolution	100
			of the Core Structure	100
		3.3.7	Filament with Axial Core Structure Variation	121
		3.3.8	Vortices in a Background Flow with Order One Vorticity	124
		3.3.9	Motion and Core Structure of a Geostrophic Vortex	129
	3.4	Vortex	Filaments with Compressible Core Structure	134
		3.4.1	Evolution Equations for Compressible Vortex	195
			Filaments	130
		3.4.2	Outer Solution for a Compressible Filament	130
		3.4.3	Inner Solution for a Compressible Filament	138
		3.4.4	The Leading-Order Equations	143
		3.4.5	The First Order Equations and the velocity	1 4 4
		0.4.0	of the Filament	144
		3.4.6	The Second-Order Equations and the Evolution	150
		047	of the Core Structure	190
		3.4.7	Evolution Equations for the Compressible Core	151
			Structure	101
4.	Nor	nlinear	Dynamics of Nearly Straight Vortex Filaments.	155
	4.1	Self-St	tretching and Curvature Nonlinearity	158
		4.1.1	The Klein-Majda Regime	158
		4.1.2	Linearization of Nonlocal Self-Induction	
			in the Klein-Majda Regime	159
		4.1.3	Hasimoto's Transformation	161
		4.1.4	Interaction of Curvature and Self-Induction Effects	168
		4.1.5	Structure of Solutions	175
	4.2	Linear	r and Nonlinear Stability of a Filament	
		in an	Imposed Straining Field	179
		4.2.1	Derivation of the Filament Equation	
			with External Strain	182
		4.2.2	Rigorous Nonlinear Stability Theory	183
		4.2.3	Linear Stability Theory	185
		4.2.4	Numerical Solutions for the Filament Equation	
			with External Strain	187
	4.3	Nonli	near Vortex Pair Instability	191
		4.3.1	Derivation of the Filament Pair Equations	196
		4.3.2	Linear Stability of Parallel Vortices of Opposite Strength	L
			and Crow's Theory	203
		4.3.3	Numerical Solutions for an Antisymmetric Vortex Pair	206

ţ

	4.4	Large	Amplitude and Long Wavelength Displacements of a		
		Filam	ent Pair	212	
		4.4.1	Amplitude–Wavelength Scalings	213	
		4.4.2	Nonlinear Curvature – Potential Flow Interactions	214	
		4.4.3	Mathematical Properties	216	
		4.4.4	Long-Wave Interactions of Filament Pairs	218	
5.	Nu	merica	l Simulation		
	of S	Slende	r Vortex Filaments	227	
	5.1	Validi	ity of the Slender Vortex Asymptotics and Applications.	229	
		5.1.1	Coaxial Vortex Rings in an Axisymmetric Flow	229	
		5.1.2	Interactions of Vortex Filaments	234	
	5.2	Vorte	x Element Methods for Slender Vortex Filaments	242	
		5.2.1	Thin-Tube Models for Weakly Stretched Vortices	244	
		5.2.2	Performance in the Klein-Majda Regime	249	
		5.2.3	Representation of Core Dynamics		
			in Thin-Tube Simulations	262	
		5.2.4	Enhancement of Performance	271	
6.	Numerical Simulations of the Merging				
	of '	Vortice	es or Filaments	285	
	6.1	Classi	ification of Merging Problems		
		and E	Efficient Numerical Schemes	287	
		6.1.1	Classification of Merging Problems	288	
		6.1.2	Efficient Numerical Scheme		
			and Computational Domain	290	
		6.1.3	Long-Time Behavior	294	
	6.2	Mergi	ing of Two-Dimensional Vortices	296	
		6.2.1	Numerical Simulation of Vortex Merging		
			and the Roll-Up of Thin Shear Layers	299	
		6.2.2	Rules of Merging of Vortices	. 309	
		6.2.3	Approximate Solution of Navier–Stokes Equations		
			Using Superposition of Lamb Vortices	316	
	-6.3	Mergi	ing or Intersection of Vortex Filaments	328	
		6.3.1	Merging of Coaxial Vortex Rings	329	
		6.3.2	Numerical Modeling of Merging of Vortex Filaments	339	
7.	Flo	Flow Generated			
	7.1	Sound	d Generation by an Unsteady Vortical Flow	349	
		7.1.1	Expansion Schemes and the Governing Equations	. 351	
		7.1.2	The Vortical Flow Field	. 353	
		7.1.3	The Acoustic Field	. 358	
		7.1.4	Sound Generation due to Turbulence	. 359	
		7.1.5	Sound Generation by Vortex Filaments	. 360	
	7.2	Vorti	cal Flow Outside a Sphere and Sound Generation	. 362	

		7.2.1	The Image of a Rotational Flow due to the Presence
			of a Sphere
		7.2.2	Sound Generation due to the Presence of a Sphere 365
		7.2.3	A Slender Vortex Filament Outside a Sphere
		7.2.4	Computational Examples 370
8.	Sou	nd Ge	nerated Flow
0.	8.1	Single	Time Scale, Low Mach Number Limits
	0.1	8.1.1	Expansions in a Single Time and Multiple Spacial Scales 384
		812	A Single Spatial Scale: Zero Mach Number Flow
		0,1.2	in Acoustically Compact Domains
		81.3	Multiple Spatial Scales: Long-Wave Acoustics
		0.110	and Baroclinic Small-Scale Flow
		8.1.4	Localized Small-Scale Flow and Multiple Time Scales:
		0.1.1	Thermoacoustics
	8.2	Comp	utational Schemes
		for Lo	w Mach Number Flows
		8.2.1	A Zero Mach Number Godunov-Type Scheme 400
		8.2.2	Vorticity-Based Formulation
	8.3	Appli	cation to Thermoacoustic Refrigeration
		8.3.1	Convergence Properties and Modeling Approximations 413
		8.3.2	Temperature Difference Across a Thermoacoustic
			Couple
		8.3.3	Quantitative Visualization of the Flow Around
			a Thermoacoustic Couple
		8.3.4	Analysis of Performance 431
9.	Epi	logue .	
	9.1	Nume	rical Strategies for Extended Slender
		Vorte	x Simulations
	9.2	A Mu	ltiple Scales Asymptotic Framework for Meteorological
		Mode	ling
		9.2.1	Universal Parameters and Distinguished Limits 457
		9.2.2	Nondimensionalization and General Multiscale Ansatz 459
		9.2.3	Distinguished Limits Versus Multiparameter
			Expansions
		9.2.4	Perspectives
Ар	penc	lices	
٨	Ca	ione !	~ Dought
н.	for	uich -	g Equations
	101	rngue	r-Order Solutions
в.	Sec	ond-O	rder Two-Time Solutions 469

÷.

C.	Equations of Motion of Filaments 475			
D.	Formulae for the Coefficients in $(6.2.74)$ and $(6.2.75)$ 479			
Е.	Transformations to Filament Attached Coordinates 44			
•	E.1	Conservation Laws in Orthogonal Coordinates		
	E.2	Scalar Transport Equations		
		in Orthogonal Coordinates		
	E.3	Compressible Flow Equations		
		in Filament Attached Coordinates		
	E.4	Formulas for the Coordinate System Attached to $\mathcal C$		
Re	feren	aces		
Inc	lex .			