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PART ONE

From the Uncertainty Relation to Many-Body Systems

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PART TWO

From Symmetries in Quantum Physics to Electroweak Interactions

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