

Contents

Preface	xi
I THEORY	1
1 Introduction	3
1.1 It's a Nonlinear World	3
1.2 Symbolic Computation	6
1.2.1 Examples of Maple Operations	7
1.2.2 Getting Maple Help	22
1.2.3 Use of Maple in Studying Nonlinear Physics	23
1.3 Nonlinear Experimental Activities	31
1.4 Scope of Part I (Theory)	32
2 Nonlinear Systems. Part I	35
2.1 Nonlinear Mechanics	35
2.1.1 The Simple Pendulum	35
2.1.2 The Eardrum	42
2.1.3 Nonlinear Damping	44
2.1.4 Nonlinear Lattice Dynamics	47
2.2 Competition Phenomena	50
2.2.1 Volterra–Lotka Competition Equations	50
2.2.2 Population Dynamics of Fox Rabies in Europe	55
2.2.3 Selection and Evolution of Biological Molecules	57
2.2.4 Laser Beam Competition Equations	60
2.2.5 Rapoport's Model for the Arms Race	62
2.3 Nonlinear Electrical Phenomena	64
2.3.1 Nonlinear Inductance	64
2.3.2 An Electronic Oscillator (the Van der Pol Equation)	65
2.4 Chemical and Other Oscillators	71
2.4.1 Chemical Oscillators	71
2.4.2 The Beating Heart	75
3 Nonlinear Systems. Part II	77
3.1 Pattern Formation	77
3.1.1 Chemical Waves	77
3.1.2 Snowflakes and Other Fractal Structures	79

3.1.3	Rayleigh–Bénard Convection	84
3.1.4	Cellular Automata and the Game of Life	86
3.2	Solitons	93
3.2.1	Shallow Water Waves (KdV and Other Equations)	95
3.2.2	Sine-Gordon Equation	99
3.2.3	Self-Induced Transparency	103
3.2.4	Optical Solitons	104
3.2.5	The Jovian Great Red Spot (GRS)	107
3.2.6	The Davydov Soliton	107
3.3	Chaos and Maps	108
3.3.1	Forced Oscillators	108
3.3.2	Lorenz and Rössler Systems	111
3.3.3	Poincaré Sections and Maps	113
3.3.4	Examples of One- and Two-Dimensional Maps	116
4	Topological Analysis	121
4.1	Introductory Remarks	121
4.2	Types of Simple Singular Points	125
4.3	Classifying Simple Singular Points	129
4.3.1	Poincaré’s Theorem for the Vortex (Center)	134
4.4	Examples of Phase Plane Analysis	134
4.4.1	The Simple Pendulum	134
4.4.2	The Laser Competition Equations	137
4.4.3	Example of a Higher Order Singularity	143
4.5	Bifurcations	150
4.6	Isoclines	153
4.7	3-Dimensional Nonlinear Systems	155
5	Analytic Methods	163
5.1	Introductory Remarks	163
5.2	Some Exact Methods	164
5.2.1	Separation of Variables	166
5.2.2	The Bernoulli Equation	170
5.2.3	The Riccati Equation	172
5.2.4	Equations of the Structure $d^2y/dx^2 = f(y)$	175
5.3	Some Approximate Methods	186
5.3.1	Maple Generated Taylor Series Solution	187
5.3.2	The Perturbation Approach: Poisson’s Method	188
5.3.3	Lindstedt’s Method	195
5.4	The Krylov–Bogoliubov (KB) Method	203
5.5	Ritz and Galerkin Methods	208
6	The Numerical Approach	215
6.1	Finite-Difference Approximations	216
6.2	Euler and Modified Euler Methods	218
6.2.1	Euler Method	219
6.2.2	The Modified Euler Method	223
6.3	Rungé–Kutta (RK) Methods	229

6.3.1	The Basic Approach	229
6.3.2	Examples of Common RK Algorithms	232
6.4	Adaptive Step Size	237
6.4.1	A Simple Example	237
6.4.2	The Step Doubling Approach	240
6.4.3	The RKF 45 Algorithm	241
6.5	Stiff Equations	244
6.6	Implicit and Semi-Implicit Schemes	248
7	Limit Cycles	255
7.1	Stability Aspects	255
7.2	Relaxation Oscillations	263
7.3	Bendixson's First Theorem	267
7.3.1	Bendixson's Negative Criterion	267
7.3.2	Proof of Theorem	267
7.3.3	Applications	269
7.4	The Poincaré–Bendixson Theorem	270
7.4.1	Poincaré–Bendixson Theorem	271
7.4.2	Application of the Theorem	271
7.5	The Brusselator Model	274
7.5.1	Prigogine–Lefever (Brusselator) Model	274
7.5.2	Application of the Poincaré–Bendixson Theorem	275
7.6	3-Dimensional Limit Cycles	280
8	Forced Oscillators	283
8.1	Duffing's Equation	283
8.1.1	The Harmonic Solution	286
8.1.2	The Nonlinear Response Curves	288
8.2	The Jump Phenomenon and Hysteresis	294
8.3	Subharmonic & Other Periodic Oscillations	297
8.4	Power Spectrum	305
8.5	Chaotic Oscillations	312
8.6	Entrainment and Quasiperiodicity	324
8.6.1	Entrainment	324
8.6.2	Quasiperiodicity	326
8.7	The Rössler and Lorenz Systems	328
8.7.1	The Rössler Attractor	328
8.7.2	The Lorenz Attractor	330
8.8	Hamiltonian Chaos	331
8.8.1	Hamiltonian Formulation of Classical Mechanics	331
8.8.2	The Hénon-Heiles Hamiltonian	333
9	Nonlinear Maps	343
9.1	Introductory Remarks	343
9.2	The Logistic Map	344
9.2.1	Introduction	344
9.2.2	Geometrical Representation	346
9.3	Fixed Points and Stability	351

9.4	The Period-Doubling Cascade to Chaos	354
9.5	Period Doubling in the Real World	357
9.6	The Lyapunov Exponent	360
9.7	Stretching and Folding	363
9.8	The Circle Map	366
9.9	Chaos versus Noise	371
9.10	2-Dimensional Maps	376
9.10.1	Introductory Remarks	376
9.10.2	Classification of Fixed Points	378
9.10.3	Delayed Logistic Map	379
9.10.4	Mandelbrot Map	380
9.11	Mandelbrot and Julia Sets	382
9.12	Nonconservative versus Conservative Maps	384
9.13	Controlling Chaos	386
9.14	3-Dimensional Maps: Saturn's Rings	391
10	Nonlinear PDE Phenomena	401
10.1	Introductory Remarks	401
10.2	Burgers' Equation	402
10.3	Bäcklund Transformations	410
10.3.1	The Basic Idea	410
10.3.2	Examples	410
10.3.3	Nonlinear Superposition	413
10.4	Solitary Waves	416
10.4.1	The Basic Approach	416
10.4.2	Phase Plane Analysis	417
10.4.3	KdV Equation	421
10.4.4	Sine-Gordon Equation	428
10.4.5	The Three-Wave Problem	430
11	Numerical Simulation	437
11.1	Finite Difference Approximations	437
11.2	Explicit Methods	442
11.2.1	Diffusion Equation	442
11.2.2	Fisher's Nonlinear Diffusion Equation	451
11.2.3	Klein-Gordon Equation	452
11.2.4	KdV Solitary Wave Collisions	455
11.3	Von Neumann Stability Analysis	458
11.3.1	Linear Diffusion Equation	458
11.3.2	Burgers' Equation	459
11.4	Implicit Methods	461
11.5	Method of Characteristics	464
11.5.1	Colliding Laser Beams	464
11.5.2	General Equation	467
11.5.3	Sine-Gordon Equation	469
11.6	Higher Dimensions	471

12 Inverse Scattering Method	473
12.1 Lax's Formulation	474
12.2 Application to KdV Equation	476
12.2.1 Direct Problem	476
12.2.2 Time Evolution of the Scattering Data	479
12.2.3 The Inverse Problem	481
12.3 Multi-Soliton Solutions	482
12.4 General Input Shapes	485
12.5 The Zakharov-Shabat/AKNS Approach	487
II EXPERIMENTAL ACTIVITIES	493
Introduction to Nonlinear Experiments	495
1 Spin Toy Pendulum	499
2 Driven Eardrum	503
3 Nonlinear Damping	507
4 Anharmonic Potential	511
5 Iron Core Inductor	517
6 Nonlinear LRC Circuit	521
7 Tunnel Diode Negative Resistance Curve	527
8 Tunnel Diode Self-Excited Oscillator	533
9 Forced Duffing Equation	537
10 Focal Point Instability	543
11 Compound Pendulum	549
12 Stable Limit Cycle	551
13 Van der Pol Limit Cycle	559
14 Relaxation Oscillations: Neon Bulb	563
15 Relaxation Oscillations: Drinking Bird	569
16 Relaxation Oscillations: Tunnel Diode	573
17 Hard Spring	577
18 Nonlinear Resonance Curve: Mechanical	581

19 Nonlinear Resonance Curve: Electrical	585
20 Nonlinear Resonance Curve: Magnetic	589
21 Subharmonic Response: Period Doubling	593
22 Diode: Period Doubling	595
23 Five-Well Magnetic Potential	599
24 Power Spectrum	605
25 Entrainment and Quasiperiodicity	609
26 Quasiperiodicity	611
27 Chua's Butterfly	613
28 Route to Chaos	617
29 Driven Spin Toy	621
30 Mapping	623
Bibliography	627
Index	641