

# CONTENTS

<b>PREFACE</b>	xxi
<b>Chapter 1 INTRODUCTION</b>	1
1.1 Cash Flows	2
1.2 Investments and Markets	3
The Comparison Principle	4
Arbitrage	4
Dynamics	5
Risk Aversion	5
1.3 Typical Investment Problems	6
Pricing	6
Hedging	7
Risk Assessment and Management	8
Pure Investment	8
Other Problems	9
1.4 Organization of the Book	9
Deterministic Cash Flow Streams	9
Single-Period Random Cash Flow Streams	10
Derivative Assets	10
General Cash Flow Streams	11
<b>Part I: DETERMINISTIC CASH FLOW STREAMS</b>	
<b>Chapter 2 THE BASIC THEORY OF INTEREST</b>	15
2.1 Principal and Interest	15
Simple Interest	15
Compound Interest	16
Compounding at Various Intervals	17
Continuous Compounding	18
Debt	19
Money Markets	19
2.2 Present Value	20
2.3 Present and Future Values of Streams	21
The Ideal Bank	21
Future Value	21
Present Value	22

	Frequent and Continuous Compounding	23
	Present Value and an Ideal Bank	23
2.4	Internal Rate of Return	24
2.5	Evaluation Criteria	26
	Net Present Value	27
	Internal Rate of Return	28
	Discussion of the Criteria	28
2.6	Applications and Extensions*	30
	Net Flows	30
	Cycle Problems	31
	Taxes	33
	Inflation	34
2.7	Summary	36
	Exercises	37
	References	41
<b>Chapter 3</b>	<b>FIXED-INCOME SECURITIES</b>	42
3.1	The Market for Future Cash	43
	Savings Deposits	43
	Money Market Instruments	44
	U.S. Government Securities	44
	Other Bonds	45
	Mortgages	46
	Annuities	46
3.2	Value Formulas	46
	Perpetual Annuities	47
	Finite-Life Streams	48
	Running Amortization*	50
	Annual Worth*	51
3.3	Bond Details	52
	Quality Ratings	53
3.4	Yield	54
	Qualitative Nature of Price–Yield Curves	55
	Other Yield Measures	58
3.5	Duration	59
	Interest Duration	60
	Macaulay Duration	60
	Explicit Formula*	61
	Qualitative Properties of Duration*	61
	Duration and Sensitivity	62
	Duration of a Portfolio	64
3.6	Immunization	65
3.7	Convexity*	68
3.8	Summary	69
	Exercises	71
	References	74

<b>Chapter 4 THE TERM STRUCTURE OF INTEREST RATES</b>	76
4.1 The Yield Curve	76
4.2 The Term Structure	78
Spot Rates	78
Discount Factors and Present Value	79
Determining the Spot Rate	81
4.3 Forward Rates	82
4.4 Term Structure Explanations	85
Expectations Theory	85
Liquidity Preference	86
Market Segmentation	87
Discussion	87
4.5 Expectations Dynamics	88
Spot Rate Forecasts	88
Discount Factors	89
Short Rates	90
Invariance Theorem	91
4.6 Running Present Value	92
4.7 Floating-Rate Bonds	95
4.8 Duration	96
Fisher-Weil Duration	96
Discrete-Time Compounding*	97
4.9 Immunization	98
4.10 Summary	100
Exercises	102
References	106
<b>Chapter 5 APPLIED INTEREST RATE ANALYSIS</b>	107
5.1 Capital Budgeting	108
Independent Projects	108
Interdependent Projects*	111
5.2 Optimal Portfolios	113
The Cash Matching Problem	114
5.3 Dynamic Cash Flow Processes	116
Representation of Dynamic Choice	117
Cash Flows in Graphs	119
5.4 Optimal Management	120
Running Dynamic Programming	120
Examples	123
5.5 The Harmony Theorem*	128
5.6 Valuation of a Firm*	130
Dividend Discount Models	130
Free Cash Flow*	132
5.7 Summary	134
Exercises	136
References	139

**Part II: SINGLE-PERIOD RANDOM CASH FLOWS**

<b>Chapter 6 MEAN–VARIANCE PORTFOLIO THEORY</b>	143
6.1 Asset Return	144
Short Sales	144
Portfolio Return	146
6.2 Random Variables	147
Expected Value	148
Variance	149
Several Random Variables	150
Covariance	150
Variance of a Sum	152
6.3 Random Returns	152
Mean–Standard Deviation Diagram	155
6.4 Portfolio Mean and Variance	156
Mean Return of a Portfolio	156
Variance of Portfolio Return	156
Diversification*	157
Diagram of a Portfolio	159
6.5 The Feasible Set	161
The Minimum-Variance Set and the Efficient Frontier	162
6.6 The Markowitz Model	164
Solution of the Markowitz Problem*	165
Nonnegativity Constraints*	168
6.7 The Two-Fund Theorem*	168
6.8 Inclusion of a Risk-Free Asset	171
6.9 The One-Fund Theorem	173
Solution Method*	173
Explicit Solution	175
6.10 Summary	175
Exercises	176
References	179
<b>Chapter 7 THE CAPITAL ASSET PRICING MODEL</b>	180
7.1 Market Equilibrium	180
7.2 The Capital Market Line	182
7.3 The Pricing Model	184
Betas of Common Stocks	187
Beta of a Portfolio	187
7.4 The Security Market Line	187
Systematic Risk	189
7.5 Investment Implications	190
7.6 Performance Evaluation	191
7.7 CAPM as a Pricing Formula	194
Linearity of Pricing and the Certainty Equivalent Form	196
7.8 Project Choice*	198

7.9	Projection Pricing	200
	Minimum Norm Pricing*	202
7.10	Correlation Pricing	203
7.11	Summary	206
	Exercises	207
	References	211
<b>Chapter 8 OTHER PRICING MODELS</b>		213
8.1	Introduction	213
8.2	Factor Models	213
	Single-Factor Model	214
	Portfolio Parameters	215
	Multifactor Models	219
	Selection of Factors	219
8.3	The CAPM as a Factor Model	220
	The Characteristic Line	221
8.4	Arbitrage Pricing Theory*	223
	Simple Version of APT	223
	Well-Diversified Portfolios	225
	General APT	226
	APT and CAPM	227
8.5	Projection Pricing with Factors	227
8.6	A Multiperiod Fallacy	229
8.7	Summary	230
	Exercises	232
	References	234
<b>Chapter 9 DATA AND STATISTICS</b>		235
9.1	Basic Estimation Methods	235
	Period-Length Effects	236
	Mean Blur	238
9.2	Estimation of Other Parameters	240
	Estimation of $\sigma$	240
	$\alpha$ Blur	241
9.3	The Effect of Estimation Errors	242
	Three Views	243
	Maximum Tangent	245
	Compounding Effect	248
9.4	Conservative Approaches	248
	Better Estimates*	249
9.5	Tilting Away From Equilibrium*	250
9.6	Summary	252
	Exercises	253
	References	255
<b>Chapter 10 RISK MEASURES</b>		257
10.1	Value at Risk	258

Properties of VaR	260
Capital Requirement	260
10.2 Computation of Value at Risk	261
Model-Based Method	261
Other Models	264
Shortcut for Discrete Distributions	264
Empirical Approach for Market Risk*	265
10.3 Criticisms of VaR	266
Diversification Failure	266
Poor Assessment of Risk	267
Discontinuous Value	268
10.4 Coherent Risk Measures	269
10.5 Conditional Value at Risk	270
10.6 Coherent Characterization*	272
10.7 Convexity*	274
10.8 Summary	275
Exercises	275
References	277
<b>Chapter 11 GENERAL PRINCIPLES</b>	279
11.1 Introduction	279
11.2 Utility Functions	279
Equivalent Utility Functions	281
11.3 Risk Aversion	282
Derivatives	284
Risk Aversion Coefficients	284
Certainty Equivalent	284
11.4 Specification of Utility Functions*	285
Direct Measurement of Utility	285
Parameter Families	287
Questionnaire Method	288
11.5 Utility Functions and the Mean–Variance Criterion*	288
Quadratic Utility	288
Normal Returns	290
11.6 Linear Pricing	291
Type A Arbitrage	291
Portfolios	292
Type B Arbitrage	292
11.7 Portfolio Choice	293
11.8 Arbitrage Bounds	296
11.9 Zero-Level Pricing	297
11.10 Log-Optimal Pricing*	299
11.11 Finite State Models	301
Completeness	302
State Prices	302
Positive State Prices	302

11.12 Risk-Neutral Pricing	304
11.13 Summary	306
Exercises	308
References	311

**Part III: DERIVATIVE SECURITIES**

<b>Chapter 12 FORWARDS, FUTURES, AND SWAPS</b>	315
12.1 Pricing Principles	316
12.2 Forward Contracts	318
Forward Interest Rates	319
12.3 Forward Prices	319
Costs of Carry	322
Tight Markets	324
Investment Assets	325
12.4 The Value of a Forward Contract	326
12.5 Swaps*	327
Value of a Commodity Swap	327
Value of an Interest Rate Swap	329
12.6 Basics of Futures Contracts	329
12.7 Futures Prices	332
12.8 Relation to Expected Spot Price*	335
12.9 The Perfect Hedge	336
12.10 The Minimum-Variance Hedge	336
12.11 Optimal Hedging*	340
12.12 Hedging Nonlinear Risk*	341
12.13 Summary	345
Exercises	346
References	349
<b>Chapter 13 MODELS OF ASSET DYNAMICS</b>	350
13.1 Binomial Lattice Model	351
13.2 The Additive Model	353
Normal Price Distribution	354
13.3 The Multiplicative Model	355
Lognormal Prices	355
Real Stock Distributions	356
13.4 Typical Parameter Values*	357
13.5 Lognormal Random Variables	358
13.6 Random Walks and Wiener Processes	359
Generalized Wiener Processes and Ito Processes	361
13.7 A Stock Price Process	362
Lognormal Prices	363
Standard Ito Form	363
Simulation	365
13.8 Ito's Lemma*	366

13.9 Binomial Lattice Revisited	368
13.10 Summary	370
Exercises	370
References	373
<b>Chapter 14 BASIC OPTIONS THEORY</b>	
14.1 Option Concepts	375
14.2 The Nature of Option Values	377
Time Value of Options	379
Other Factors Affecting the Value of Options	379
14.3 Option Combinations and Put–Call Parity	380
Put–Call Parity	381
14.4 Early Exercise	382
14.5 Single-Period Binomial Options Theory	383
14.6 Multiperiod Options	386
No Early Exercise*	389
14.7 More General Binomial Problems	389
Put Options	389
Dividend and Term Structure Problems*	391
Futures Options*	391
14.8 Evaluating Real Investment Opportunities	393
Real Options	397
Linear Pricing	399
14.9 General Risk-Neutral Pricing*	401
14.10 Three-principle Power	402
Decomposition of the Pricing Principles	403
14.11 Summary	403
Exercises	404
References	408
<b>Chapter 15 ADDITIONAL OPTIONS TOPICS</b>	
15.1 Introduction	410
15.2 The Black–Scholes Equation	410
Proof of the Black–Scholes Equation*	412
Self-Financing Strategies*	414
15.3 Call Option Formula	414
15.4 Risk-Neutral Valuation*	416
15.5 Delta	417
15.6 Replication, Synthetic Options, and Portfolio Insurance*	419
15.7 Volatility Smiles	422
Equality of Implied Volatilities	423
Risk-Neutral Probability Density*	424
15.8 Computational Methods	425
Monte Carlo Simulation	426
Finite-Difference Methods	427
Binomial and Trinomial Lattices	429

15.9 Exotic Options	431
Pricing*	433
15.10 Comparison of Methods	434
15.11 Storage Costs and Dividends*	435
Binomial Form	435
Brownian Motion Form*	436
15.12 Martingale Pricing*	437
15.13 Axioms and Black–Scholes	438
Market Price of Risk	440
15.14 Summary	440
Exercises	442
References	446
<b>Chapter 16 INTEREST RATE DERIVATIVES</b>	448
16.1 Examples of Interest Rate Derivatives	448
16.2 The Need for a Theory	450
16.3 The Binomial Approach	451
Implied Term Structure	452
No Arbitrage Opportunities	454
16.4 Pricing Applications	455
Bond Derivatives	455
Forwards and Futures*	455
Futures*	457
16.5 Leveling and Adjustable-Rate Loans*	457
Adjustable-Rate Loans	458
16.6 The Forward Equation	461
16.7 Matching the Term Structure	464
The Ho–Lee Model	464
The Black–Derman–Toy Model	465
Matching Implied Volatilities	465
16.8 Immunization	467
16.9 Collateralized Mortgage Obligations*	469
16.10 Models of Interest Rate Dynamics*	473
16.11 Continuous-Time Solutions*	474
The Backward Equation	475
Affine Processes*	476
Risk-Neutral Pricing Formula	477
16.12 Extensions	477
16.13 Summary	478
Exercises	479
References	482
<b>Chapter 17 CREDIT RISK</b>	483
17.1 The Classic Merton Model	484
Probability of Default	486
Credit Spread	486

17.2	First Passage Times	487
	Lattice Methods	488
	Early Default*	490
	Coupons*	491
17.3	Rating Methods	492
17.4	Intensity (Reduced-Form) Model	493
	Poisson Processes	493
	Inhomogeneous Process	495
17.5	Stochastic Intensity Model*	495
17.6	Intermediate Receipts	496
17.7	Analytically Tractable Cox Processes	497
	Model Fitting	497
17.8	Simulation	498
	Direct Simulation	498
	A Better Way	499
17.9	Lattice Methods	500
17.10	Correlated Defaults	503
17.11	Credit Derivatives	505
	Bonds and Loans	506
	Credit Default Swaps (CDS's)	506
	Forwards and Options on CDS's	508
	Total Return Swaps (TRS's)	508
	Collateralized Debt Obligations (CDO's)	509
17.12	Summary	511
	Exercises	512
	References	513

#### **Part IV: GENERAL CASH FLOW STREAMS**

<b>Chapter 18 OPTIMAL PORTFOLIO GROWTH</b>	517	
18.1	The Investment Wheel	517
	Analysis of the Wheel	519
18.2	The Log Utility Approach to Growth	519
	Log Utility Form	521
	Examples	521
18.3	Properties of the Log-Optimal Strategy*	525
18.4	Alternative Approaches*	526
	Other Utility	526
18.5	Continuous-Time Growth	528
	Dynamics of Several Stocks	528
	Portfolio Dynamics	529
	Implications for Growth	530
	The Portfolio of Maximum Growth Rate	530
18.6	The Feasible Region	531

The Efficient Frontier	531
Inclusion of a Risk-Free Asset	532
18.7 The Log-Optimal Pricing Formula*	536
Market Data	539
18.8 Log-Optimal Pricing and the Black–Scholes Equation*	540
18.9 Summary	541
Exercises	542
References	546
 <b>Chapter 19 GENERAL INVESTMENT EVALUATION</b>	
19.1 General Present Value	547
Projects and Opportunities	548
19.2 Multiperiod Securities*	548
Assets	549
Portfolio Strategies	549
Arbitrage	550
Short-Term Risk-Free Rates	550
19.3 Risk-Neutral Pricing	550
19.4 Optimal Pricing	552
The Single-Period Problem	552
Applications	553
19.5 The Double Lattice	555
19.6 Pricing in a Double Lattice	557
19.7 Investments with Private Uncertainty	560
General Approach	562
19.8 Buying Price Analysis	566
Certainty Equivalent and Exponential Utility	567
Sequential Calculation of CE	568
Multiperiod Case	569
General Approach	570
19.9 Pricing Axioms for Continuous Time	572
Option Formula	575
Risk-Neutral Form	575
Alternative Forms	575
19.10 Summary	576
Exercises	576
References	578
 <b>Appendix A BASIC PROBABILITY THEORY</b>	
A.1 General Concepts	579
A.2 Normal Random Variables	580
A.3 Lognormal Random Variables	581

<b>Appendix B CALCULUS AND OPTIMIZATION</b>	583
B.1 Functions	583
B.2 Differential Calculus	584
B.3 Optimization	585
<b>ANSWERS TO EXERCISES</b>	588
<b>INDEX</b>	594