
Contents

Preface	vii
Chapter 1. Natural Numbers	1
1.1. Peano Systems	2
1.2. Addition	4
1.3. Multiplication	9
1.4. Order	12
1.5. Isomorphism of Peano Systems	13
1.6. A Set-Theoretic Model	17
1.7. Recursion	19
1.8. Mathematical Induction	24
1.9. Algebraic Structures	27
Notes	28
Exercises	29
Chapter 2. Integers	37
2.1. Definition of the Integers	37
2.2. Addition of Integers	40
2.3. Multiplication of Integers	44
2.4. Order	47
2.5. Rings and Integral Domains	49
Notes	54
Exercises	55
Chapter 3. Rational Numbers	59
3.1. Definition of Rational Numbers	60

3.2. Addition of Rational Numbers	61
3.3. Multiplication of Rational Numbers	62
3.4. Order	64
3.5. Algebraic Structures on \mathbf{Q}	68
3.6. Convergence in an Ordered Field	71
3.7. Limitations of \mathbf{Q}	77
Notes	80
Exercises	80
Chapter 4. Real Numbers	85
4.1. Definition of Real Numbers	85
4.2. Operations on \mathbf{R}	87
4.3. \mathbf{R} as a Field	88
4.4. \mathbf{R} as an Ordered Field	90
4.5. Cauchy Completeness of \mathbf{R}	93
4.6. Dedekind Completeness of \mathbf{R}	96
4.7. Continuous Functions on \mathbf{R}	101
Notes	103
Exercises	104
Chapter 5. Complex Numbers	107
5.1. Definition of Complex Numbers	108
5.2. The Field \mathbf{C} of Complex Numbers	108
5.3. \mathbf{C} as a Vector Space	111
5.4. \mathbf{C} as a Normed Algebra	112
5.5. Convergence in \mathbf{C}	113
5.6. Roots of Complex Numbers	117
5.7. Continuous functions	118
5.8. The Fundamental Theorem of Algebra	120
Notes	122
Exercises	123
Appendix A. Sets, Relations, Functions	127
A.1. Sets	127
A.2. Operations on Sets	129
A.3. Relations	132
A.4. Functions and Operations	133
Notes	134
Exercises	135
Bibliography	139
Index	141