Contents

I Introduction 1

1 A Chronicle of System Design Limitations 3
- 1.1 Introduction . . . 3
- 1.2 Performance Limitations in Dynamical Systems . . . 6
- 1.3 Time Domain Constraints . . . 9
  - 1.3.1 Integrals on the Step Response . . . 9
  - 1.3.2 Design Interpretations . . . 13
  - 1.3.3 Example: Inverted Pendulum . . . 16
- 1.4 Frequency Domain Constraints . . . 18
- 1.5 A Brief History . . . 19
- 1.6 Summary . . . 20

Notes and References . . . 21

II Limitations in Linear Control 23

2 Review of General Concepts 25
- 2.1 Linear Time-Invariant Systems . . . 26
  - 2.1.1 Zeros and Poles . . . 27
  - 2.1.2 Singular Values . . . 29

2.1.3 Frequency Response 29
2.1.4 Coprime Factorization 30
2.2 Feedback Control Systems 31
2.2.1 Closed-Loop Stability 32
2.2.2 Sensitivity Functions 32
2.2.3 Performance Considerations 33
2.2.4 Robustness Considerations 35
2.3 Two Applications of Complex Integration 36
2.3.1 Nyquist Stability Criterion 37
2.3.2 Bode Gain-Phase Relationships 40
2.4 Summary 45
Notes and References 45

3 SISO Control **47**
3.1 Bode Integral Formulae 47
3.1.1 Bode's Attenuation Integral Theorem 48
3.1.2 Bode Integrals for S and T 51
3.1.3 Design Interpretations 59
3.2 The Water-Bed Effect 62
3.3 Poisson Integral Formulae 64
3.3.1 Poisson Integrals for S and T 65
3.3.2 Design Interpretations 67
3.3.3 Example: Inverted Pendulum 73
3.4 Discrete Systems 74
3.4.1 Poisson Integrals for S and T 75
3.4.2 Design Interpretations 78
3.4.3 Bode Integrals for S and T 79
3.4.4 Design Interpretations 81
3.5 Summary 83
Notes and References 84

4 MIMO Control **85**
4.1 Interpolation Constraints 85
4.2 Bode Integral Formulae 87
4.2.1 Preliminaries 88
4.2.2 Bode Integrals for S 91
4.2.3 Design Interpretations 96
4.3 Poisson Integral Formulae 98
4.3.1 Preliminaries 98
4.3.2 Poisson Integrals for S 99
4.3.3 Design Interpretations 102
4.3.4 The Cost of Decoupling 103
4.3.5 The Impact of Near Pole-Zero Cancelations 105

4.3.6 Examples 107
4.4 Discrete Systems 114
4.4.1 Poisson Integral for S 114
4.5 Summary 116
Notes and References 116

5 Extensions to Periodic Systems **119**
5.1 Periodic Discrete-Time Systems 119
5.1.1 Modulation Representation 120
5.2 Sensitivity Functions 123
5.3 Integral Constraints 124
5.4 Design Interpretations 127
5.4.1 Time-Invariant Map as a Design Objective 127
5.4.2 Periodic Control of Time-invariant Plant 130
5.5 Summary 132
Notes and References 132

6 Extensions to Sampled-Data Systems **135**
6.1 Preliminaries. 136
6.1.1 Signals and System 136
6.1.2 Sampler, Hold and Discretized System 137
6.1.3 Closed-loop Stability 140
6.2 Sensitivity Functions 141
6.2.1 Frequency Response 141
6.2.2 Sensitivity and Robustness 143
6.3 Interpolation Constraints 145
6.4 Poisson Integral formulae 150
6.4.1 Poisson Integral for S^0 150
6.4.2 Poisson Integral for T^0 153
6.5 Example: Robustness of Discrete Zero Shifting 156
6.6 Summary 158
Notes and References 158

III Limitations in Linear Filtering **161**

7 General Concepts **163**
7.1 General Filtering Problem 163
7.2 Sensitivity Functions 165
7.2.1 Interpretation of the Sensitivities 167
7.2.2 Filtering and Control Complementarity 169
7.3 Bounded Error Estimators 172
7.3.1 Unbiased Estimators 176

7.4 Summary 177
Notes and References 177

8 SISO Filtering **179**
8.1 Interpolation Constraints 179
8.2 Integral Constraints 181
8.3 Design Interpretations 184
8.4 Examples: Kalman Filter 189
8.5 Example: Inverted Pendulum 194
8.6 Summary 195
Notes and References 195

9 MIMO Filtering **197**
9.1 Interpolation Constraints 198
9.2 Poisson Integral Constraints 199
9.3 The Cost of Diagonalization 202
9.4 Application to Fault Detection 205
9.5 Summary 208
Notes and References 209

10 Extensions to SISO Prediction **211**
10.1 General Prediction Problem 211
10.2 Sensitivity Functions 214
10.3 BEE Derived Predictors 215
10.4 Interpolation Constraints 216
10.5 Integral Constraints 219
10.6 Effect of the Prediction Horizon 221
10.6.1 Large Values of τ 221
10.6.2 Intermediate Values of τ 222
10.7 Summary 228
Notes and References 228

11 Extensions to SISO Smoothing **229**
11.1 General Smoothing Problem 229
11.2 Sensitivity Functions 233
11.3 BEE Derived Smoothers 233
11.4 Interpolation Constraints 235
11.5 Integral Constraints 236
11.5.1 Effect of the Smoothing Lag 238
11.6 Sensitivity Improvement of the Optimal Smoother 239
11.7 Summary 243
Notes and References 244

IV Limitations in Nonlinear Control and Filtering 245

12 Nonlinear Operators 247
12.1 Nonlinear Operators 247
12.1.1 Nonlinear Operators on a Linear Space 248
12.1.2 Nonlinear Operators on a Banach Space 249
12.1.3 Nonlinear Operators on a Hilbert Space 250
12.2 Nonlinear Cancelations 251
12.2.1 Nonlinear Operators on Extended Banach Spaces . . 252
12.3 Summary 254
Notes and References 254

13 Nonlinear Control 255
13.1 Review of Linear Sensitivity Relations 255
13.2 A Complementarity Constraint 256
13.3 Sensitivity Limitations 258
13.4 The Water-Bed Effect 260
13.5 Sensitivity and Stability Robustness 262
13.6 Summary 264
Notes and References 265

14 Nonlinear Filtering 267
14.1 A Complementarity Constraint 267
14.2 Bounded Error Nonlinear Estimation 270
14.3 Sensitivity Limitations 271
14.4 Summary 273
Notes and References 273

A Review of Complex Variable Theory 275
A.1 Functions, Domains and Regions 275
A.2 Complex Differentiation 276
A.3 Analytic functions 278
A.3.1 Harmonic Functions 280
A.4 Complex Integration 281
A.4.1 Curves 281
A.4.2 Integrals 283
A.5 Main Integral Theorems 289
A.5.1 Green's Theorem 289
A.5.2 The Cauchy Integral Theorem 291
A.5.3 Extensions of Cauchy's Integral Theorem 293
A.5.4 The Cauchy Integral Formula 296
A.6 The Poisson Integral Formula 298
A.6.1 Formula for the Half Plane 298

A.6.2 Formula for the Disk 302
A.7 Power Series 304
A.7.1 Derivatives of Analytic Functions 304
A.7.2 Taylor Series 306
A.7.3 Laurent Series 309
A.8 Singularities 311
A.8.1 Isolated Singularities 311
A.8.2 Branch Points 313
A.9 Integration of Functions with Singularities 315
A.9.1 Functions with Isolated Singularities 315
A.9.2 Functions with Branch Points 319
A.10 The Maximum Modulus Principle 321
A.11 Entire Functions 322
Notes and References 325

B Proofs of Some Results in the Chapters **327**
B.1 Proofs for Chapter 4 327
B.2 Proofs for Chapter 6 334
B.2.1 Proof of Lemma 6.2.2 334
B.2.2 Proof of Lemma 6.2.4 339
B.2.3 Proof of Lemma 6.2.5 341

C The Laplace Transform of the Prediction Error **343**

D Least Squares Smoother Sensitivities for Large τ **347**

References **351**

Index **359**