

# Contents

<b>Prologue</b>	xviii
<b>1 Introduction</b>	1
1.1 The Descriptive Layers of Physical Reality	1
1.2 Units and Notations	3
1.3 Hamiltonian and Lagrangian Mechanics	4
1.3.1 Review of Variational Calculus	4
1.3.2 Noether's Theorem	6
1.3.3 Applications of Noether's Theorem	7
<b>2 Relativistic Invariance</b>	9
2.1 Introduction	9
2.2 The Three-Dimensional Rotation Group	11
2.3 Three-Dimensional Spinors	14
2.4 Three-Dimensional Spinorial Tensors	18
2.5 The Lorentz Group	20
2.6 Generators and Lie Algebra of the Lorentz Group	23
2.7 The Group $SL(2, \mathbb{C})$	25
2.8 The Four-Dimensional Spinors	27
2.9 Space Inversion and Bispinors	30
2.10 Finite-Dimensional Representations of $SU(2)$ and $SL(2, \mathbb{C})$	33
2.11 Problems	35
<b>3 The Electromagnetic Field</b>	37
3.1 Introduction	37
3.2 Tensor Formulation of Maxwell's Equations	38
3.3 Maxwell's Equations and Differential Forms	40
3.4 Choice of a Gauge	43
3.5 Invariance under Change of Coordinates	45
3.6 Lagrangian Formulation	47
3.6.1 The Euler–Lagrange Equations and Noether's Theorem	48
3.6.2 Examples of Noether Currents	51
3.6.3 Application to Electromagnetism	53
3.7 Interaction with a Charged Particle	56
3.8 Green Functions	58
3.8.1 The Green Functions of the Klein–Gordon Equation	59
3.8.2 The Green Functions of the Electromagnetic Field	63

3.9	Applications	65
3.9.1	The Liénard–Wiechert Potential	65
3.9.2	The Larmor Formula	69
3.9.3	The Thomson Formula	70
3.9.4	The Limits of Classical Electromagnetism	71
<b>4</b>	<b>General Relativity: A Field Theory of Gravitation</b>	<b>73</b>
4.1	The Equivalence Principle	73
4.1.1	Introduction	73
4.1.2	The Principle	74
4.1.3	Deflection of Light by a Gravitational Field	76
4.1.4	Influence of Gravity on Clock Synchronisation	76
4.2	Curved Geometry	77
4.2.1	Introduction	77
4.2.2	Tensorial Calculus for the Reparametrisation Symmetry	78
4.2.3	Affine Connection and Covariant Derivation	80
4.2.4	Parallel Transport and Christoffel Coefficients	83
4.2.5	Geodesics	85
4.2.6	The Curvature Tensor	88
4.3	Reparametrization Gauge Symmetry and Einstein’s General Relativity	90
4.3.1	Reparametrisation Invariance as a Gauge Symmetry	90
4.3.2	Reparametrisation Invariance and Energy–Momentum Tensor	93
4.3.3	The Einstein–Hilbert Equation	95
4.4	The Limits of Our Perception of Space and Time	98
4.4.1	Direct Measurements	99
4.4.2	Possible Large Defects	100
<b>5</b>	<b>The Physical States</b>	<b>103</b>
5.1	Introduction	103
5.2	The Principles	104
5.2.1	Relativistic Invariance and Physical States	105
5.3	The Poincaré Group	107
5.3.1	The Irreducible Representations of the Poincaré Group	107
5.3.2	The Generators of the Poincaré Group	110
5.4	The Space of the Physical States	114
5.4.1	The One-Particle States	114
5.4.2	The Two- or More Particle States without Interaction	115
5.4.3	The Fock Space	116
5.4.4	Introducing Interactions	117
5.5	Problems	118

<b>6 Relativistic Wave Equations</b>	<b>120</b>
6.1 Introduction	120
6.2 The Klein–Gordon Equation	120
6.3 The Dirac Equation	123
6.3.1 The $\gamma$ Matrices	126
6.3.2 The Conjugate Equation	127
6.3.3 The Relativistic Invariance	128
6.3.4 The Current	129
6.3.5 The Hamiltonian	129
6.3.6 The Standard Representation	129
6.3.7 The Spin	131
6.3.8 The Plane Wave Solutions	132
6.3.9 The Coupling with the Electromagnetic Field	135
6.3.10 The Constants of Motion	136
6.3.11 Lagrangian and Green Functions	137
6.4 Relativistic Equations for Vector Fields	138
<b>7 Towards a Relativistic Quantum Mechanics</b>	<b>142</b>
7.1 Introduction	142
7.2 The Klein–Gordon Equation	142
7.3 The Dirac Equation	144
7.3.1 The Non-relativistic Limit of the Dirac Equation	144
7.3.2 Charge Conjugation	146
7.3.3 PCT Symmetry	149
7.3.4 The Massless Case	150
7.3.5 Weyl and Majorana Spinors	152
7.3.6 Hydrogenoid Systems	153
7.4 Problems	159
<b>8 Functional Integrals and Probabilistic Amplitudes</b>	<b>162</b>
8.1 Introduction	162
8.2 Brief Historical Comments	163
8.3 The Physical Approach	165
8.4 The Reconstruction of Quantum Mechanics	168
8.4.1 The Quantum Mechanics of a Free Particle	169
8.4.2 A Particle in a Potential	170
8.4.3 The Schrödinger Equation	170
8.5 The Feynman Formula	173
8.5.1 The Representations of Quantum Mechanics	173
8.5.2 The Feynman Formula for Systems with One Degree of Freedom	176
8.6 The Harmonic Oscillator	180

8.7	The Bargmann Representation	186
8.7.1	The Coherent States	186
8.7.2	The Path Integral Formula in the Bargmann Space	191
8.8	Problems	194
<b>9</b>	<b>Functional Integrals and Quantum Mechanics: Formal Developments</b>	<b>196</b>
9.1	$T$ -Products	196
9.1.1	General Definition	196
9.1.2	Application to the Harmonic Oscillator	197
9.2	$S$ -Matrix and $T$ -Products	201
9.2.1	Three Examples	204
9.3	Elements of Perturbation Theory	206
9.4	Generalizations	211
9.4.1	Three-Dimensional Quantum Mechanics	211
9.4.2	The Free Scalar Field	212
9.5	Problems	217
<b>10</b>	<b>The Euclidean Functional Integrals</b>	<b>218</b>
10.1	Introduction	218
10.1.1	The Wiener Measure	220
10.2	The Gaussian Measures in Euclidean Field Theories	224
10.2.1	Definition	225
10.2.2	The Integration by Parts Formula	228
10.2.3	The Wick Ordering	229
10.3	Application to Interacting Fields	230
10.3.1	The 2-Point Function	231
10.3.2	The 4-Point Function	236
10.3.3	The General Feynman Rules	238
10.4	Problems	239
<b>11</b>	<b>Fermions and Functional Formalism</b>	<b>240</b>
11.1	Introduction	240
11.2	The Grassmann Algebras	243
11.2.1	The Derivative	243
11.2.2	The Integration	244
11.3	The Clifford Algebras	249
11.4	Fermions in Quantum Mechanics	250
11.4.1	Quantum Mechanics and Fermionic Oscillators	250
11.4.2	The Free Fermion Fields	253
11.5	The Path Integrals	254
11.5.1	The Case of Quantum Mechanics	254
11.5.2	The Case of Field Theory	257

<b>12 Relativistic Quantum Fields</b>	<b>260</b>
12.1 Introduction	260
12.2 Relativistic Field Theories	260
12.2.1 The Axiomatic Field Theory	261
12.3 The Asymptotic States	279
12.3.1 Introduction	279
12.3.2 The Fock Space	281
12.3.3 Existence of Asymptotic States	282
12.4 The Reduction Formulae	286
12.4.1 The Feynman Diagrams	292
12.5 The Case of the Maxwell Field	296
12.5.1 The Classical Maxwell Field	296
12.5.2 The Quantum Field: I. The Functional Integral	298
12.5.3 The Quantum Field: II. The Particle Concept	300
12.5.4 The Casimir Effect	304
12.6 Quantization of a Massive Field of Spin-1	305
12.7 The Reduction Formulae for Photons	308
12.8 The Reduction Formulae for Fermions	309
12.9 Quantum Electrodynamics	310
12.9.1 The Feynman Rules	310
12.10 A Formal Expression for the $S$ -Matrix	311
12.11 Problems	319
<b>13 Applications</b>	<b>320</b>
13.1 On Cross Sections	320
13.2 Formal Theory of Scattering in Quantum Mechanics	324
13.2.1 An Integral Equation for the Green Function	325
13.2.2 The Cross Section in Quantum Mechanics	335
13.3 Scattering in Field Theories	340
13.3.1 The Case of Two Initial Particles	341
13.3.2 The Case of One Initial Particle	343
13.4 Applications	344
13.5 The Feynman Rules for the $S$ -Matrix	353
13.5.1 Feynman Rules for Other Theories	354
13.6 Problems	358
<b>14 Geometry and Quantum Dynamics</b>	<b>360</b>
14.1 Introduction. QED Revisited	360
14.2 Non-Abelian Gauge Invariance and Yang–Mills Theories	362
14.3 Field Theories of Vector Fields	365
14.4 Gauge Fixing and BRST Invariance	369
14.4.1 Introduction	369
14.4.2 The Traditional Faddeev–Popov Method	369

14.4.3	Graded Notation for the Classical and Ghost Yang–Mills Fields	376
14.4.4	Determination of the BRST Symmetry as the Extension of the Gauge Symmetry for the Classical and Ghost Fields	378
14.4.5	General BRST Invariant Action for the Yang–Mills Theory	382
14.5	Feynman Rules for the BRST Invariant Yang–Mills Action	385
14.6	BRST Quantization of Gravity Seen as a Gauge Theory	386
14.7	The Gribov Ambiguity: The Failure of the Gauge-Fixing Process beyond Perturbation Theory	389
14.7.1	A Simple Example	389
14.7.2	The Gribov Question in a Broader Framework	390
14.8	Historical Notes	393
14.9	Problems	396
<b>15</b>	<b>Broken Symmetries</b>	<b>398</b>
15.1	Introduction	398
15.2	Global Symmetries	399
15.2.1	An Example from Classical Mechanics	399
15.2.2	Spontaneous Symmetry Breaking in Non-relativistic Quantum Mechanics	400
15.2.3	A Simple Field Theory Model	403
15.2.4	The Linear $\sigma$ -Model	405
15.2.5	The Non-linear $\sigma$ -Model	408
15.2.6	Goldstone Theorem	410
15.3	Gauge Symmetries	412
15.3.1	The Abelian Model	413
15.3.2	The Non-Abelian Case	416
15.4	Problems	418
<b>16</b>	<b>Quantum Field Theory at Higher Orders</b>	<b>420</b>
16.1	Existence of Divergences in Loop Diagrams. Discussion	420
16.2	Connected and 1-PI Diagrams	421
16.3	Power Counting. Definition of Super-Renormalisable, Renormalizable, and Non-renormalisable Quantum Field Theories	425
16.4	Regularisation	428
16.5	Renormalisation	433
16.5.1	1-Loop Diagrams	433
16.5.2	Some 2-Loop Examples	441
16.5.3	All Orders	443
16.5.4	An ‘Almost’ Renormalisable Theory	446
16.5.5	Composite Operators	448
16.6	The Renormalisation Group	450

16.6.1	General Discussion	450
16.6.2	The Renormalisation Group in Dimensional Regularisation	453
16.6.3	Dependence of the $\beta$ and $\gamma$ Functions on the Renormalization Scheme	456
16.7	Problems	457
<b>17</b>	<b>A First Glance at Renormalisation and Symmetry</b>	<b>462</b>
17.1	Introduction	462
17.2	Global Symmetries	463
17.3	Gauge Symmetries: Examples	468
17.3.1	The Adler–Bell–Jackiw Anomaly	468
17.3.2	A Path Integral Derivation	471
17.3.3	The Axial Anomaly and Renormalisation	476
17.3.4	A Consistency Condition for Anomalies	476
17.4	The Breaking of Conformal Invariance	478
17.5	A Non-Perturbative Anomaly	482
17.6	Problems	485
<b>18</b>	<b>Renormalisation of Yang–Mills Theory and BRST Symmetry</b>	<b>486</b>
18.1	Introduction	486
18.2	Generating Functional of BRST Covariant Green Functions	487
18.2.1	BRST Ward Identities in a Functional Form	489
18.3	Anomaly Condition	490
18.3.1	General Solution for the Anomalies of the Ward Identities	491
18.3.2	The Possible Anomalous Vertices and the Anomaly Vanishing Condition	495
18.4	Dimensional Regularisation and Multiplicative Renormalisation	497
18.4.1	Introduction	497
18.4.2	Linear Gauges and Ward Identities for the BRST Symmetry and Ghost Equations of Motion	498
18.4.3	Inverting the Ward Identities in Linear Gauges for a Local Field and Source Functional	499
18.4.4	The Structure of the Counter-terms within the Dimensional Regularisation Method	501
18.5	Observables	504
18.6	Problems	505
<b>19</b>	<b>Some Consequences of the Renormalisation Group</b>	<b>507</b>
19.1	Introduction	507
19.2	The Asymptotic Behaviour of Green Functions	508
19.3	Stability and the Renormalization Group	511
19.4	Dimensional Transmutation	514
19.5	Problems	518

<b>20 Analyticity Properties of Feynman Diagrams</b>	521
20.1 Introduction	521
20.2 Singularities of Tree Diagrams	522
20.3 Loop Diagrams	524
20.4 Unstable Particles	528
20.5 Cutkosky Unitarity Relations	532
20.6 The Analytic $S$ -Matrix Theory	534
20.7 Problems	543
<b>21 Infrared Singularities</b>	544
21.1 Introduction. Physical Origin	544
21.2 The Example of Quantum Electrodynamics	545
21.3 General Discussion	550
21.4 Infrared Singularities in Other Theories	551
21.5 Problems	554
<b>22 Coherent States and Classical Limit of Quantum Electrodynamics</b>	555
22.1 Introduction	555
22.2 The Definition of Coherent States	556
22.3 Fluctuations	559
22.3.1 Time Evolution of Coherent States	560
22.3.2 Dispersion of Coherent States	560
22.4 Coherent States and the Classical Limit of QED towards Maxwell Theory	561
22.5 Squeezed States	563
22.6 Problems	565
<b>23 Quantum Field Theories with a Large Number of Fields</b>	566
23.1 Introduction	566
23.2 Vector Models	567
23.3 Fields in the Adjoint Representation	571
23.4 The Large $N$ Limit as a Classical Field Theory	574
23.5 Problems	578
<b>24 The Existence of Field Theories beyond the Perturbation Expansion</b>	580
24.1 Introduction	580
24.2 The Equivalence between Relativistic and Euclidean Field Theories	582
24.3 Construction of Field Theories	584
24.4 The Zero-Dimensional $\lambda\phi^4$ Model	590
24.4.1 The Divergence of the Perturbation Series	591
24.4.2 The Borel Summability	592
24.5 General Facts about Scalar Field Theories in $d = 2$ or $d = 3$ Dimensions	594
24.6 The $\lambda\phi^4$ Theory in $d = 2$ Dimensions	597



24.6.1	The Divergence of the $\lambda\phi_2^4$ Perturbation Series	597
24.6.2	The Existence of the $\lambda\phi_2^4$ Theory	601
24.6.3	The Cluster Expansion	607
24.6.4	The Mayer Expansion	617
24.6.5	The Infinite Volume Limit of $\lambda\phi_2^4$	621
24.6.6	The Borel Summability of the $\lambda\phi_2^4$ Theory	622
24.6.7	The Mass Gap for $\phi_2^4$ in a Strong External Field	623
24.7	The $\lambda(\phi)^4$ Theory in $d = 3$ Dimensions	625
24.7.1	The Expansion: Definition	630
24.7.2	The Expansion Completed	632
24.7.3	The Results	634
24.8	The Massive Gross–Neveu Model in $d = 2$ Dimensions	635
24.8.1	Definition of the Model	635
24.8.2	The Infinite Volume Limit	639
24.8.3	The Removal of the Ultraviolet Cut-off	639
24.8.4	The Behaviour of the Effective Constants and the Approximate Renormalization Group Flow	640
24.9	The Yang–Mills Field Theory in $d = 4$	644
24.9.1	A Physical Problem	644
24.9.2	Many Technical Problems	645
<b>25</b>	<b>Fundamental Interactions</b>	<b>648</b>
25.1	Introduction. What Is an ‘Elementary Particle’?	648
25.2	The Four Interactions	649
25.3	The Standard Model of Weak and Electromagnetic Interactions	651
25.3.1	A Brief Summary of the Phenomenology	651
25.3.2	Model Building	655
25.3.3	The Lepton World	655
25.3.4	Extension to Hadrons	660
25.3.5	The Neutrino Masses	668
25.3.6	Some Sample Calculations	671
25.3.7	Anomalies in the Standard Model	676
25.4	A Gauge Theory for Strong Interactions	680
25.4.1	Are Strong Interactions Simple?	680
25.4.2	Quantum Chromodynamics	683
25.4.3	Quantum Chromodynamics in Perturbation Theory	692
25.4.4	Quantum Chromodynamics on a Space–Time Lattice	712
25.4.5	Instantons	729
25.5	Problems	740
<b>26</b>	<b>Beyond the Standard Model</b>	<b>746</b>
26.1	Why	747
26.1.1	The Standard Model Has Been Enormously Successful	747

26.1.2	Predictions for New Physics	748
26.1.3	Unsolved Problems of the Standard Model	752
26.2	Grand Unified Theories	752
26.2.1	Generalities	752
26.2.2	The Simplest GUT: $SU(5)$	754
26.2.3	Dynamics of GUTs	757
26.2.4	Other Grand Unified Theories	764
26.2.5	Magnetic Monopoles	769
26.3	The Trial of Scalars	780
<b>27</b>	<b>Supersymmetry, or the Defence of Scalars</b>	<b>784</b>
27.1	Introduction	784
27.2	The Supersymmetry Algebra	785
27.3	Why This Particular Algebra; or All Possible Supersymmetries of the $S$ Matrix	786
27.4	Representations in Terms of One-Particle States	787
27.4.1	Massive Case	787
27.4.2	Massless Case	789
27.5	Representations in Terms of Field Operators: Superspace	791
27.6	A Simple Field Theory Model	797
27.7	Supersymmetry and Gauge Invariance	803
27.7.1	The Abelian Case	803
27.7.2	The Non-Abelian Case	806
27.7.3	Extended Supersymmetries	808
27.8	Spontaneous Symmetry Breaking and Supersymmetry	811
27.8.1	Goldstone and BEH Phenomena in the Presence of Supersymmetry	812
27.8.2	Spontaneous Supersymmetry Breaking in Perturbation Theory	818
27.8.3	Dynamical Breaking of Supersymmetry	821
27.9	Dualities in Supersymmetric Gauge Theories	823
27.10	Twisted Supersymmetry and Topological Field Theories	833
27.10.1	Introduction	833
27.10.2	A Quantum Mechanical Toy Model	837
27.10.3	Yang–Mills TQFT	839
27.11	Supersymmetry and Particle Physics	848
27.11.1	Supersymmetry and the Standard Model	851
27.11.2	Supersymmetry and Grand Unified Theories	854
27.11.3	The Minimal Supersymmetric Standard Model	855
27.12	Gauge Supersymmetry	858
27.12.1	$N=1$ Supergravity	859
27.12.2	$N = 8$ Supergravity	861
27.13	Problems	862

<b>Appendix A</b>	<b>Tensor Calculus</b>	<b>863</b>
A.1	Algebraic Theory of Tensors	863
A.1.1	Definitions	863
A.1.2	Examples	864
A.1.3	Algebraic Properties of Tensors	866
A.1.4	Bases	866
A.2	Manifolds and Tensors	868
A.2.1	Manifolds, Tangent, and Cotangent Bundles	868
A.2.2	Differential of a Mapping	869
A.2.3	Vector Fields	870
A.2.4	Cotangent Bundle	871
A.2.5	Tensors	871
A.2.6	Lie Derivative	875
A.2.7	Riemannian Structure	877
<b>Appendix B</b>	<b>Differential Calculus</b>	<b>879</b>
B.1	Differential Form	879
B.2	Exterior Differential	883
B.2.1	Integration	884
<b>Appendix C</b>	<b>Groups and Lie Algebras</b>	<b>889</b>
C.1	Lie Groups	889
C.1.1	Definitions	889
C.1.2	Representations	890
C.1.3	Lie Groups	892
C.1.4	One Parameter Subgroup. Tangent Space	893
C.2	Lie Algebras	896
C.2.1	Definition	896
C.2.2	Matrix Lie Algebras	898
<b>Appendix D</b>	<b>A Collection of Useful Formulae</b>	<b>902</b>
D.1	Units and Notations	902
D.2	Free Fields	903
D.3	Feynman Rules for Scattering Amplitudes	906
D.4	Examples	910
<b>Appendix E</b>	<b>Extract from Maxwell's <i>A Treatise on Electricity and Magnetism</i></b>	<b>912</b>
<b>Index</b>		<b>915</b>