

CONTENTS

<i>Lecturers</i>	ix
<i>Participants</i>	xi
<i>Préface (French)</i>	xv
<i>Preface (English)</i>	xix
<i>Course 1. Lectures on Directed Paths in Random Media, by M. Kardar</i>	1
1. Introduction	5
2. High temperature expansions for the Ising model	7
3. Characteristic functions and cumulants	9
4. The one dimensional chain	12
5. Directed paths and the transfer matrix	16
6. Moments of the correlation function	21
7. The probability distribution in two dimensions	25
8. Higher dimensions	28
9. Random signs	31
10. Other realizations of DPRM	35
11. Quantum interference of strongly localized electrons	36
12. The locator expansion and forward scattering paths	39
13. Magnetic field response	42
14. Unitary propagation	46
15. Unitary averages	48
16. Summing all paths in high dimensions	53
17. The Ising model on a square lattice	59
18. Singular behavior	64
19. The two dimensional spin glass	66
20. Results for the two dimensional spin glass	69
References	71

<i>Course 2. Quantization of Geometry, by J. Ambjørn</i>	77
1. Introduction	81
2. Bosonic propagators and random paths	84
2.1. Quantization	84
2.2. One-dimensional gravity	87
2.3. Scaling relations	90
2.4. Smooth random walks	93
2.5. Fermionic random walks	100
3. Random surfaces and strings	105
3.1. Definition of the model	105
3.2. Physical observable	115
3.3. Non-scaling of the string tension	123
3.4. Branched polymers	126
3.5. Extrinsic curvature terms (I)	128
3.6. Supersymmetric random surfaces	133
4. Matrix models and two-dimensional quantum gravity	135
4.1. Matrix models	135
4.2. The loop equations	143
4.3. Complete solution to leading order in $1/N^2$	148
4.4. The scaling limit	150
4.5. Generalizations	153
5. The mystery of $c > 1$	156
5.1. The Ising model	156
5.2. Multiple Ising spins	158
5.3. Random surfaces with extrinsic curvature (II)	164
6. Euclidean quantum gravity in $d > 2$	166
6.1. Basic questions in Euclidean quantum gravity	166
6.2. Definition of simplicial quantum gravity for $d > 2$	169
6.3. Observables	174
6.4. Branched polymers	179
6.5. Numerical simulations	180
6.6. Results	183
7. Discussion	190
References	191

<i>Course 3. Amphiphilic Membranes, by L. Peliti</i>	195
1. Introduction	199
2. Amphiphilic molecules and the phases they form	200
3. Isolated membranes: the Helfrich hamiltonian	212
4. Vesicle shapes	223
5. Shape fluctuations in vesicles	236
6. Interacting fluid membranes	246

Appendix A. Differential equations for vesicle shapes	274
Appendix B. The Faddeev–Popov determinant	276
Appendix C. One-loop calculation of the renormalization group flow	278
Appendix D. The Liouville model	280
References	283
<i>Course 4. What is String Theory?, by J. Polchinski</i>	287
1. Conformal field theory	292
1.1. The operator product expansion	292
1.2. Ward identities	297
1.3. Conformal invariance	299
1.4. Mode expansions	301
1.5. States and operators	306
1.6. Other CFT's	310
1.7. Other algebras	316
1.8. Riemann surfaces	319
1.9. CFT on Riemann surfaces	322
2. String theory	327
2.1. Why strings?	327
2.2. String basics	330
2.3. The spectrum	333
2.4. The Weyl anomaly	337
2.5. BRST quantization	341
2.6. Generalizations	346
2.7. Interactions	349
2.8. Trees and loops	354
3. Vacua and dualities	359
3.1. CFT's and vacua	359
3.2. Compactification on a circle	362
3.3. More on R -duality	365
3.4. $N = 0$ in $N = 1$ in ...?	368
3.5. S -Duality	370
4. String field theory or not string field theory	377
4.1. String field theory	377
4.2. Not string field theory	381
4.3. High energy and temperature	387
5. Matrix models	391
5.1. $D = 2$ String theory	391
5.2. The $D = 1$ matrix model	393
5.3. Matrix model \leftrightarrow string	397
5.4. General issues	401
5.5. Tree-level scattering	404

Contents

5.6. Spacetime gravity in the $D = 2$ string	406
5.7. Spacetime gravity in the matrix model	409
5.8. Strong nonlinearities	413
5.9. Conclusion	417
References	417
Course 5. <i>Defects in Superfluids, Superconductors and Membranes, by D.R. Nelson</i>	423
1. Introduction	427
2. Two-dimensional superfluids and superconductors	430
2.1. Superfluid helium films: experimental facts	430
2.2. Theoretical background	431
2.3. Vortex statistical mechanics and renormalization of ρ_s	437
2.4. Renormalization group and universal jump in the superfluid density	439
2.5. Two-dimensional superconductors	443
2.5.1. Naive theory	443
2.5.2. Real superconducting films	447
3. Defects in membranes and monolayers	449
3.1. Landau theory and elasticity of tethered membranes	451
3.2. Defects in monolayers	453
3.3. Defects in crystalline membranes	460
3.3.1. Disclinations and dislocations	461
3.3.2. Other defects in crystalline membranes	463
3.4. Defects in hexatic and liquid membranes	466
Appendix A. Superfluid density and momentum correlations	472
References	475
Seminar 1. <i>Some Simple (Integrable) Models of Fractional Statistics, by D. Bernard</i>	479
1. Haldane's fractional statistics	483
1.1. Definition	483
1.2. Anyon-inspired examples	484
1.3. The Bethe ansatz and fractional statistics	486
1.4. Thermodynamics	489
2. Long-range interacting models	490
3. Algebraic solution of the long-range interacting models	494
3.1. Algebraic Bethe ansatz and Yangians	494
3.2. Quantization of the spectral parameter	497
3.3. Application to the Haldane-Shastry spin chain	500
References	502

<i>Course 6. Lectures on 2D Yang–Mills Theory, Equivariant Cohomology and Topological Field Theories, by S. Cordes, G. Moore and S. Ramgoolam</i>	505
1. Introduction	513
1.1. Part I: Yang–Mills as a string theory	513
1.2. Related topics not discussed	516
1.3. Some related reviews	516
2. Introduction to Part II: general remarks on topological field theories	517
2.1. Cohomological field theories	517
2.2. Detailed overview	519
2.3. Paradigm	520
3. Equivariant cohomology	522
3.1. Classifying spaces	522
3.2. Characteristic classes	524
3.3. Weil algebra	526
3.4. Equivariant cohomology of manifolds	528
3.5. Other formulations of equivariant cohomology	530
3.6. Example 1. S^1 -equivariant cohomology	532
3.7. Example 2. G -equivariant cohomology of A	533
3.8. Equivariant cohomology vs. Lie-algebra cohomology	534
3.9. Equivariant cohomology and twisted $N = 2$ supersymmetry	536
3.10. Equivariant cohomology and symplectic group actions	538
4. Intersection numbers and their integral representations	538
4.1. Thom class and Euler class	539
4.2. Universal Thom class	542
4.3. Mathai–Quillen representative of the Thom class	543
4.4. Integral representation: antighosts	545
4.5. Integral representation: Q-invariance	546
4.6. Cartan model representative	547
4.7. Other integral representations of U	548
4.8. Dependence on choices and “BRST decoupling”	549
4.9. Superspace and “physical notation”	550
4.10. The localization principle	551
4.11. Partition functions	556
4.12. Localization and integration of equivariant differential forms	557
5. Supersymmetric quantum mechanics	560
5.1. Action and supersymmetry	561
5.2. MQ interpretation of SQM	562
5.3. Canonical quantization	564
5.4. Appendix 1. The two-dimensional supersymmetric nonlinear sigma model	566
5.5. Appendix 2. Index theorems for some elliptic operators	570

Contents

5.6.	Appendix 3. Noncompact X and runaway vacua	573
5.7.	Appendix 4. Canonical quantization with $W \neq 0$: Morse theory	574
6.	Topological sigma models	577
6.1.	Fields and equations	577
6.2.	Differential forms on $\text{MAP}(\Sigma, X)$	578
6.3.	Vector bundle	578
6.4.	Choice of connection	579
6.5.	BRST complex	581
6.6.	Gauge fermion and action	582
6.7.	Observables	582
6.8.	Correlation functions	584
6.9.	Quantum cohomology	586
6.10.	Relation to the physical sigma model	587
6.11.	Canonical approach	591
6.12.	Appendix. Examples of moduli spaces of holomorphic maps	591
7.	Topological theories with local symmetry	592
7.1.	Projection and localization	592
7.2.	The projection form	595
7.3.	A BRST construction of $\Phi(P \rightarrow M)$ for principal bundles	596
7.4.	Assembling the pieces: no gauge fixing	602
7.5.	Faddeev–Popov gauge fixing	606
7.6.	The general construction of cohomological field theory	609
8.	Topological Yang–Mills theory	611
8.1.	Basic data	611
8.2.	Equations	612
8.3.	Vector bundles	612
8.4.	Connection on V_+ and E_+	613
8.5.	BRST complex	613
8.6.	Lagrangian	616
8.7.	$D = 2$: flat connections	618
8.8.	Observables	619
8.9.	Correlation functions	624
8.10.	The canonical formulation: Floer homology	626
8.11.	Relation to “physical” Yang–Mills: $D = 4$	626
8.12.	Relation to “physical” Yang–Mills: $D = 2$	630
9.	2D topological gravity	632
9.1.	Formulation 1: $G = \text{Diff}(\Sigma) \times \text{Weyl}(\Sigma)$	632
9.2.	Formulation 2: $G = \text{Diff}(\Sigma)$	635
9.3.	Formulation 3: $G = \text{Diff}(\Sigma) \times L.L.(\Sigma)$	638
9.4.	Formulation 4: $G = \widehat{\text{Diff}(\Sigma)}$	639
9.5.	Observables	640
9.6.	Hamiltonian approach	642
9.7.	Correlation functions	642
10.	Topological string theory	643

Contents

10.1. Basic data	643
10.2. Equations	643
10.3. Antighost bundle	643
10.4. Connection on antighost bundle	644
10.5. BRST complex	645
10.6. Lagrangian	645
10.7. Index	646
10.8. Example. Riemann surface target	647
11. YM_2 as a topological string theory	647
11.1. Extending the fieldspace	648
11.2. Equations	649
11.3. BRST complex	650
11.4. Lagrangian	650
11.5. Localization	650
11.6. Nonchiral case	651
11.7. Perturbing by the area	652
11.8. Horava's theory	654
12. Euler character theories	654
13. Four dimensions: a conjecture	656
14. Conclusion	657
Appendix A. Background from differential geometry	657
A.1. Differential forms	657
A.2. Bundles	658
A.3. Connections on vector bundles and principal bundles	663
A.4. Curvature and holonomy	667
A.5. Yang–Mills equations and action	670
References	672
 <i>Seminar 2. 2D Principal Chiral Field at Large N as a Possible Solvable 2D String Theory, by V.A. Kazakov</i>	683
1. Introduction	687
2. Ground state energy and beta-function of the PCF at large N	690
3. Physical consequences of the exact solution: weak and strong coupling limits	691
4. Sketch of the solution	693
5. Conclusions	697
References	698
 <i>Course 7. Lectures on Black Holes, by A. Strominger</i>	699
1. Introduction	703
2. Causal structure and Penrose diagrams	705

Contents

2.1.	Minkowski space	705
2.2.	1 + 1 Dimensional Minkowski space	708
2.3.	Schwarzschild black holes	708
2.4.	Gravitational collapse and the Vaidya spacetimes	710
2.5.	Event horizons, apparent horizons and trapped surfaces	711
3.	Black holes in two dimensions	713
3.1.	General relativity in the S-wave sector	713
3.2.	Classical dilaton gravity	714
3.3.	Eternal black holes	715
3.4.	Coupling to conformal matter	717
3.5.	Hawking radiation and the trace anomaly	718
3.6.	The quantum state	720
3.7.	Including the back-reaction	722
3.8.	The large N approximation	723
3.9.	Conformal invariance and generalizations of dilaton gravity	726
3.10.	The soluble RST model	728
4.	The information puzzle in four dimensions	733
4.1.	Can the information come out before the endpoint?	734
4.2.	Low-energy effective descriptions of the Planckian endpoint	739
4.3.	Remnants?	740
4.4.	Information destruction?	744
4.5.	The superposition principle	746
4.6.	Energy conservation	749
4.7.	The new rules	752
4.8.	Superselction sectors, α -parameters, and the restoration of unitarity	754
5.	Conclusions and outlook	756
	References	758

<i>Seminar 3. Black Hole Evaporation and Complementarity, by E. Verlinde</i>	763
--	-----

References	770
------------	-----

<i>Course 8. Part I. Quantum Theory of Large Systems of Non-Relativistic Matter, by J. Fröhlich, U.M. Studer and E. Thiran</i>	771
--	-----

1.	Introduction	775
1.1.	Sources, and acknowledgements	775
1.2.	Topics not treated in these notes	776
2.	The Pauli equation and its symmetries	777
2.1.	Gauge-invariant form of the Pauli equation	778

Contents

2.2. Aharonov–Bohm effect	780
2.3. Aharonov–Casher effect	781
3. Gauge invariance in non-relativistic quantum many-particle systems	783
3.1. Differential geometry of the background	784
3.2. Systems of spinning particles coupled to external electromagnetic and geometric fields	789
3.3. Moving coordinates and quantum-mechanical Larmor theorem	795
4. Some key effects related to the $U(1) \times SU(2)$ gauge invariance of non-relativistic quantum mechanics	804
4.1. “Tidal” Aharonov–Bohm and “Geometric” Aharonov–Casher effects	804
4.2. Flux quantization	806
4.3. Barnett and Einstein–de Haas Effects	807
4.4. Meissner–Ochsenfeld effect and London theory of superconductivity	808
4.5. Quantum Hall effect	810
5. Scaling limit of the effective action of Fermi systems, and classification of states of non-relativistic matter	818
6. Scaling limit of the effective action of a two-dimensional, incompressible quantum fluid	828
6.1. Scaling limit of the effective action	829
6.2. Linear response theory and current sum rules	839
6.3. Quasi-particle excitations and a spin-singlet electron pairing mechanism	844
6.3.1. Laughlin vortices and fractional statistics	844
6.3.2. Spinon quantum mechanics	846
6.3.3. A spin-singlet electron pairing mechanism	849
7. Anomaly cancellation and algebras of chiral edge currents in two-dimensional, incompressible quantum fluids	852
7.1. Integer quantum Hall effect and edge currents	853
7.2. Edge excitations in spin-polarized quantum Hall fluids	861
8. Classification of incompressible quantum Hall fluids	869
8.1. QH fluids and QH lattices: basic concepts	870
8.1.1. Chiral quantum Hall lattices (CQHLs)	872
8.2. A dictionary between the physics of QH fluids and the mathematics of QH lattices	874
8.3. Basic invariants of chiral QH lattices (CQHLs) and their physical interpretations	880
8.4. General theorems and classification results for CQHLs	884
8.4.1. Shift maps and their implications	888
8.5. Maximally symmetric CQHLs	892
8.5.1. Classification	896
8.6. Summary and physical implications of the classification results	897
8.6.1. General structuring results	897
8.6.2. Theoretical implications versus experimental data	899
References	907

<i>Course 8. Part II. Renormalization Group Methods: Landau–Fermi Liquid and BCS Superconductor, by T. Chen, J. Fröhlich and M. Seifert</i>	913
1. Background material	920
1.1. Thermodynamics and quantum statistical mechanics	920
1.2. Systems of identical particles	923
1.3. Functional integrals: bosons	926
1.4. Functional integrals: fermions	928
2. Weakly coupled electron gases	930
2.1. Free electrons and dimensional reduction	930
2.2. Weakly coupled electrons and the renormalization group (RG)	932
3. The renormalization group flow	936
3.1. Scaling of action and fields	936
3.2. Integrating out modes	942
3.3. The BCS channel	951
4. Spontaneous breaking of gauge invariance, and superconductivity	958
<i>Seminar 4. A Rigorous Analysis of the Superconducting Phase of an Electron–Phonon System, by J. Feldman, J. Magnen, V. Rivasseau and E. Trubowitz</i>	971
1. Introduction	975
2. The “First Regime”	979
3. The “Second Regime”	986
4. The “Third Regime”	987
References	993
<i>Course 9. Scaling and Selection in Cellular Structures and Living Polymers, by D. Mukamel</i>	995
1. Two dimensional cellular structures	999
2. Living polymers	1005
References	1008
<i>Course 10. Geometrical Properties of Loops and Cluster Boundaries, by J. Cardy</i>	1011
1. Introduction	1015

Contents

2. Area of loops	1017
3. Finite-length loops	1021
3.1. Radius of gyration	1022
4. Calculation of k , c and ν'	1023
References	1026
<i>Course 11. Exact Resummations in the Theory of Hydrodynamic Turbulence: 0. Line-Resummed Diagrammatic Perturbation Approach, by V. L'vov and I. Procaccia</i>	1027
1. Introduction	1031
2. Naive Perturbation Theory	1035
2.1. Introduction	1035
2.2. Naive Perturbation Theory for the velocity field	1039
2.3. Naive perturbation theory for statistical quantities	1041
2.3.1. Statistics	1041
2.3.2. The mean velocity	1043
2.3.3. The Green's function	1044
2.3.4. The 2-point velocity correlation function	1045
3. Resummations	1048
3.1. The resummation of the mean velocity	1048
3.2. The Dyson resummation for the Green's function	1050
3.3. The Wyld resummation for the 2-point correlation function	1052
3.4. Line resummation	1054
3.5. Intuitive meaning of the Dyson and Wyld equations	1057
3.6. Three and higher order velocity correlation functions	1060
4. Functional Integral Formulation	1060
4.1. Introduction	1060
4.2. Setting up the generating functional	1061
4.3. The evaluation of the bare propagators	1066
4.4. Diagrammatic expansion	1069
5. Suggestions for further reading	1072
Appendix A. The Jacobian	1073
References	1074

<i>Course 12. Random Systems and Replica Field Theory, by M. Mézard</i>	1077
---	------

1. Introduction	1081
2. Manifolds in random media	1084

Contents

3. Thermal fluctuations without disorder	1084
4. Random forces	1085
5. Random potential: variational approach	1085
6. Physical interpretation of the solution	1088
References	1089

<i>Seminars by Participants</i>	1091
---------------------------------	------