

Contents

0	Introduction	1
1	Fundamental Ideas	4
1.1	Variational Principles	4
1.1.1	Geometrical optics—reflection and refraction	6
1.1.2	The scope of variational principles	9
1.2	Euclidean Space and Time	10
1.3	Partial Derivatives	14
1.4	e , π and Gaussian Integrals	18
1.4.1	Radioactive decay	19
1.4.2	Waves and periodic functions	21
1.4.3	The Gaussian integral	21
1.4.4	The method of steepest descents	24
2	Motions of Bodies—Newton's Laws	26
2.1	Introduction	26
2.2	Newton's Laws of Motion	27
2.3	The Principle of Least Action	29
2.3.1	Motion in one dimension	31
2.3.2	A simple example and a simple method	32
2.3.3	Motion in a general potential and Newton's second law	35
2.3.4	The calculus of variations	36
2.3.5	The unimportance of the endpoints	38
2.4	The Motion of Several Bodies and Newton's Third Law	39
2.5	Motion of One Body in Three Dimensions	42
2.5.1	The harmonic oscillator	43
2.6	Central Forces	44
2.6.1	Circular orbits	46
2.7	The Attractive Inverse Square Law Force	48
2.8	G and the Mass of the Earth	52
2.9	Composite Bodies and Centre of Mass Motion	53
2.10	The Kepler 2-Body Problem	56
2.10.1	Binary stars	57
2.11	Lagrangian Points	58
2.12	Conservation of Energy	63
2.13	Friction and Dissipation	65

3 Fields—Maxwell’s Equations	67
3.1 Fields	67
3.2 The Scalar Field Equation	69
3.3 Waves	72
3.4 Divergence and Curl	74
3.5 Electromagnetic Fields and Maxwell’s Equations	75
3.5.1 What Maxwell’s equations tell us	78
3.6 Electrostatic Fields	81
3.6.1 Charge and dipole moment	84
3.7 Electromagnetic Waves	86
3.8 Magnetostatics	90
3.9 Principle of Least Action for Electromagnetic Fields	92
3.10 The Lorentz Force	93
3.10.1 The Lorentz force from the principle of least action	97
3.11 Field Energy and Momentum	98
3.12 Dynamics of Particles and Fields	100
4 Special Relativity	103
4.1 Introduction	103
4.2 Lorentz Transformations	105
4.3 Relativistic Dynamics	110
4.3.1 Comparison of Newtonian and relativistic dynamics	114
4.3.2 $E = mc^2$	116
4.4 More on 4-Vectors	117
4.5 The Relativistic Character of Maxwell’s Equations	118
4.6 Relativistic Principles of Least Action	122
5 Curved Space	126
5.1 Spherical Geometry	126
5.1.1 Geodesics	127
5.2 Non-Euclidean, Hyperbolic Geometry	128
5.3 Gaussian Curvature	129
5.4 Riemannian Geometry	132
5.4.1 Simple examples of metrics	134
5.5 Tensors	137
5.5.1 Covariant derivatives and Christoffel symbols	140
5.5.2 Christoffel symbols in plane polar coordinates	143
5.6 The Riemann Curvature Tensor	143
5.6.1 Riemann curvature in plane polar coordinates	145
5.6.2 Riemann curvature on a sphere	145
5.6.3 The 3-sphere	146
5.7 The Geodesic Equation	146
5.7.1 Geodesics in plane polar coordinates	149
5.7.2 The equation of geodesic deviation	149
5.8 Applications	152

6 General Relativity	158
6.1 The Equivalence Principle	158
6.2 The Newtonian Gravitational Field and Tidal Forces	159
6.3 Minkowski Space	162
6.4 Curved Spacetime Geometry	164
6.4.1 Weak gravitational fields	166
6.5 The Gravitational Field Equation	167
6.5.1 The energy-momentum tensor	168
6.5.2 The Einstein tensor and the Einstein equation	170
6.5.3 Determining the constant of proportionality	172
6.6 The Classic Tests of General Relativity	173
6.6.1 The perihelion advance of Mercury	173
6.6.2 The deflection of starlight	174
6.6.3 Clocks and gravitational redshift	174
6.7 The Schwarzschild Solution of the Einstein Equation	177
6.7.1 The Newtonian limit	179
6.8 Particle Motion in Schwarzschild Spacetime	180
6.9 Light Deflection in Schwarzschild Spacetime	183
6.10 The Interior Schwarzschild Solution	185
6.11 Black Holes	187
6.11.1 Eddington–Finkelstein coordinates	189
6.11.2 The Kerr metric	191
6.12 Gravitational Waves	195
6.12.1 The detection of gravitational waves	196
6.13 The Einstein–Hilbert Action	199
7 Quantum Mechanics	203
7.1 Introduction	203
7.2 Position and Momentum in Quantum Mechanics	205
7.3 The Schrödinger Equation	207
7.3.1 The free particle	210
7.3.2 The harmonic oscillator	212
7.4 Interpretation of Wavefunctions—Observables	214
7.4.1 Position probabilities	215
7.4.2 Other physical quantities—hermitian operators	216
7.4.3 Measurements of observables	218
7.5 Expectation Values	220
7.6 After a Measurement	222
7.7 Uncertainty Relations	222
7.8 Scattering and Tunnelling	225
7.9 Variational Principles in Quantum Mechanics	227
8 Quantum Mechanics in Three Dimensions	231
8.1 Introduction	231
8.2 Position and Momentum Operators	232
8.2.1 Particle in a box	234
8.3 Angular Momentum Operators	235

8.3.1	Eigenfunctions of \hat{l}^2 using Cartesian coordinates	237
8.4	The Schrödinger Equation with a Spherical Potential	240
8.4.1	The Coulomb potential	241
8.4.2	Spectroscopy	242
8.5	Spin	244
8.5.1	The Stern-Gerlach experiment	245
8.5.2	The Zeeman effect	246
8.5.3	Other spin representations	247
8.6	Spin $\frac{1}{2}$ as a Quantum Paradigm	248
8.7	Quantum Mechanics of Several Identical Particles	249
8.7.1	The Fermi sphere	254
8.8	Bosons, Fermions and Spin	255
8.9	Return to the Action	256
9	Atoms, Molecules and Solids	261
9.1	Atoms	261
9.1.1	Atomic orbitals	263
9.1.2	Atomic shell model	264
9.2	Molecules	267
9.2.1	Covalent bonding	267
9.2.2	Polar bonds	274
9.2.3	Simple molecules	276
9.3	Organic Chemistry	277
9.3.1	Hückel theory—benzene	277
9.3.2	Polyenes	280
9.4	Solids	283
9.4.1	Covalent solids	284
9.5	Band Theory	286
9.5.1	Atomic lattices	286
9.5.2	Bloch's theorem	289
9.5.3	Bloch states in a finite crystal	290
9.5.4	The tight-binding model	291
9.5.5	The nearly free electron model	293
9.5.6	Ionic solids	294
9.5.7	Example of caesium chloride	294
9.5.8	Metals	297
9.5.9	Example of copper	298
9.6	Ferromagnetism	300
10	Thermodynamics	303
10.1	Introduction	303
10.1.1	What is heat?	304
10.1.2	The ideal gas law	304
10.1.3	The microscopic origin of heat	306
10.1.4	Iced tea	306
10.2	Entropy and Temperature	307
10.3	The First Law of Thermodynamics	311

10.3.1 New variables	313
10.4 Subsystems—The Gibbs Distribution	315
10.5 The Maxwell Velocity Distribution	318
10.6 Ideal Gases—Equation of State and Entropy	320
10.7 Non-Ideal Gases	322
10.8 The Chemical Potential	323
10.9 Fermion and Boson Gases at Low Temperature	324
10.9.1 The Fermi–Dirac function	325
10.9.2 Pressure of a degenerate electron gas	326
10.9.3 The heat capacity of an electron gas	327
10.9.4 The Bose–Einstein function	329
10.10 Black Body Radiation	334
10.11 Lasers	337
10.12 Magnetization in Spin Systems	341
10.13 A Little about Phase Transitions	343
10.14 Hawking Radiation	347
11 Nuclear Physics	351
11.1 The Birth of Nuclear Physics	351
11.2 The Strong Force	352
11.2.1 The nuclear potential	354
11.2.2 Nucleon pairing	357
11.2.3 The liquid drop model	358
11.3 The Nuclear Shell Model	362
11.3.1 The atomic shell analogy	362
11.3.2 The harmonic oscillator	363
11.3.3 Spin–orbit coupling	365
11.3.4 Beta decay	369
11.3.5 The Nilsson model	370
11.4 Alpha Decay	372
11.5 Fission	377
11.6 Fusion	380
11.6.1 Thermonuclear fusion	381
11.6.2 Controlled nuclear fusion	385
11.7 The Island of Stability	385
11.8 Exotic Nuclei	387
11.9 Pions, Yukawa Theory and QCD	389
12 Particle Physics	393
12.1 The Standard Model	393
12.1.1 Fundamental particles	394
12.2 Quantum Field Theory	395
12.2.1 Quantizing the electromagnetic field	397
12.2.2 The quantized scalar Klein–Gordon field	398
12.3 The Dirac Field	400
12.3.1 The Dirac equation	400
12.3.2 Quantizing the Dirac field—particles and antiparticles	403

12.4 Actions and Interactions	406
12.4.1 Quantum electrodynamics	406
12.4.2 Feynman diagrams	408
12.5 The Strong Force	410
12.5.1 Quarks	413
12.5.2 Confinement	415
12.6 QCD	417
12.6.1 Gluons	417
12.6.2 Lattice QCD	421
12.6.3 Heavy quarks and exotic hadrons	422
12.7 The Weak Force	422
12.7.1 Parity violation	424
12.8 The Theory of the Electroweak Force	426
12.8.1 The Higgs mechanism	427
12.8.2 Fermion masses	429
12.8.3 Discovering the W and Z bosons and the Higgs boson	432
12.8.4 Quark mixing	434
12.8.5 How many generations?	435
12.9 Neutrino Oscillations	437
13 Stars	443
13.1 The Sun	443
13.2 The Herzsprung–Russell Diagram	444
13.3 The Birth of Stars	447
13.3.1 Stellar composition	447
13.3.2 The virial theorem	448
13.3.3 Star formation	451
13.4 Stellar Structure	453
13.4.1 The structure functions	455
13.4.2 The mass–luminosity relationship	457
13.4.3 The density–temperature relationship	458
13.5 Nucleosynthesis	458
13.5.1 The proton–proton chain	459
13.5.2 The CNO cycle	461
13.5.3 The mass–radius relationship	463
13.5.4 The mass–temperature relationship	463
13.5.5 Minimum mass of main sequence stars	463
13.5.6 The temperature–luminosity relationship	464
13.6 Giant Stars Beyond the Main Sequence	465
13.6.1 The triple alpha process	466
13.7 Late Evolution	467
13.7.1 White dwarfs	467
13.7.2 Gravitational collapse of massive stars	470
13.8 Neutron Stars	474
13.8.1 Pulsars	476
13.9 Supernovae	477

13.9.1 Gamma-ray bursts	482
13.10 The Density–Temperature Diagram	483
14 Cosmology	487
14.1 Einstein’s Universe	487
14.2 The Distance–Redshift Relationship	487
14.3 Friedmann–Robertson–Walker Cosmology	489
14.3.1 Einstein’s equation and the FRW metric	490
14.3.2 The general FRW cosmological solutions	493
14.4 Cosmological Redshift	496
14.5 Newtonian Interpretation of the FRW Cosmology	497
14.6 The Big Bang	498
14.6.1 The age of the universe	499
14.7 Dark Matter	500
14.8 The Cosmic Microwave Background	502
14.8.1 Precision measurements of the CMB	503
14.9 The Cosmological Constant	504
14.10 Galaxy Formation	505
14.11 The Inflationary Universe	507
14.11.1 Particle horizons	508
14.11.2 Inflation	510
15 Frontiers of Physics	514
15.1 The Interpretation of Quantum Mechanics	514
15.1.1 Schrödinger’s cat and Wigner’s friend	515
15.1.2 The many-worlds interpretation	518
15.1.3 The EPR paradox	518
15.1.4 The Aspect experiments	519
15.2 The Problem of Point Particles	524
15.2.1 Solitons	525
15.2.2 Skyrmions	527
15.3 Critique of the Standard Model	529
15.4 Topology and the Standard Model	530
15.5 Beyond the Standard Model	532
15.5.1 Grand Unified Theories	533
15.5.2 Supersymmetry	534
15.6 String Theory	535
15.6.1 Compactification	538
Picture Credits	541
Index	543
Biographies	557