ENCYCLOPEDIA OF MATHEMATICS AND ITS APPLICATIONS

Solving Polynomial Equation Systems

Volume IV: Buchberger Theory and Beyond

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Contents

Adieu *page* ix

PART SEVEN: Beyond 1

46 Zacharias 3

46.1 Buchberger–Zacharias Theory 5

46.2 Kandri-Rody–Kapur: Strong Gröbner Bases over a Euclidean Domain 27

46.3 Pan: Strong Gröbner Bases over a Principal Ideal Domain 31

46.4 Zacharias Rings 37

46.5 Möller : Gröbner Bases over Zacharias Rings 42

46.6 Möller: Gröbner Bases over a Principal Ideal Ring 48

46.7 Useless S-pairs and Gebauer–Möller Set in Principal Ideal Rings 50

46.8 Möller: Structural Theorem for Gröbner Bases over a Principal Ideal Ring 54

46.9 Lazard's Structural Theorem over a Principal Ideal Ring 56

46.10 Ore Conditions 63

46.11 Ore Theory 68

46.12 Lazard's Structural Theorem for Ore Extensions over a Principal Ideal Domain 83

46.13 Multivariate Ore Extensions 85

46.14 Ideal of Vanishing Polynomial 96

47 Bergman 100

47.1 Möller Left Lifting Theorem for Free Associative Algebras 102

47.2 Left Gröbner Bases for Free Associative Algebras 107

47.3 Möller's Right Lifting Theorem and Right Gröbner Bases for Free Associative Algebras 115

47.4 Bilateral Gröbner Bases: The Lifting Theorem 117

47.5 Bilateral Gröbner Bases: Pritchard's Module 121

47.6 Bilateral Gröbner Bases of an Ideal in a Free Monoid Rings over a Field 124

	47.7	Pritchard's Procedure	144
	47.8	La Scala–Levandovskyy: Letterplace Ideals	148
	47.9	Gröbner Basis for Weaker Algebras	157
	47.10	Green: Path Algebras	164
	47.11	Reinert: Buchberger Theory on Function Rings	172
	47.12	de Graaf–Wisliceny–Gerritzen: Gröbner Bases on Magma Algebras	184
48	**Ufnarovski**		191
	48.1	Ufnarovski Graph	192
	48.2	Ufnarovski: Finite-State Automata and Infinite Gröbner Basis Computation	198
	48.3	2-Variable Graph	206
	48.4	Shirayanagi: Word Tree	207
	48.5	Anick's Resolution	212
	48.6	Borges: Elimination Orderings and Symmetric Groups	217
	48.7	Taxonomy of Term Orderings	221
	48.8	Ideal Theoretical Operation	246
49	**Weispfenning**		256
	49.1	Apel–Lassner: Lie Algebras and Buchberger Theory	258
	49.2	Bergman: Diamond Lemma	261
	49.3	Skew and Solvable Polynomial Rings	264
	49.4	Kredel Conditions	268
	49.5	Kandri-Rody–Weispfenning: Solvable Polynomial Rings	271
	49.6	Twisted Polynomial Rings	278
	49.7	Structural Theorem for Bilateral Ideals of a Twisted Polynomial Ring	284
	49.8	Gateva's Monoid	288
	49.9	Gateva: Noetherianity of Finitely Presented Algebras	291
	49.10	Gateva: Binomial Skew Polynomial Rings	298
	49.11	Weispfenning Rings	314
	49.12	La Scala–Levandovskyy: Ore Extension of Multivariate Polynomial Rings	324
	49.13	La Scala: Buchberger Theory for Partial Difference Ideals	332
50	**Spear 2**		335
	50.1	Zacharias' Canonical Representation	336
	50.2	Term Ordering	349
	50.3	Pseudovaluations	351
	50.4	Gröbner Bases and Buchberger Theory	359
	50.5	Möller's Lifting Theorem	363
	50.6	Spear's Theorem	371
	50.7	Pseudovaluations (2)	377
	50.8	Toward Buchberger's Algorithm	381
	50.9	Buchberger's Algorithms	383
	50.10	The Arithmetic of the Associated Graded Ring	386

	50.11	Noetherianity	394
	50.12	Canonical Forms	397
	50.13	Case Studies	399
51	**Weispfenning II**		423
	51.1	Comprehensive Gröbner Bases	425
	51.2	Montes' Algorithm	429
	51.3	Canonical Comprehensive Gröbner Bases	436
	51.4	Wibmer: Regular Functions	441
	51.5	Kapur *et al.*: Branching via Solving Techniques	444
	51.6	Montes–Wibmer: Canonical Gröbner Cover	446
	51.7	Von Neumann Regular Rings	449
	51.8	Gröbner Bases over Von Neumann Regular Rings	454
	51.9	Comprehensiveness of Gröbner Bases over Von Neumann Regular Rings (1)	457
	51.10	Suzuki–Sato: Alternative Comprehensive Gröbner Bases	459
	51.11	Comprehensiveness of Gröbner Bases over Von Neumann Regular Rings (2)	465
52	**Sweedler**		468
	52.1	Valuation Rings	471
	52.2	Mosteing–Sweedler: Valuations and Filtrations	492
	52.3	Mosteing–Sweedler: Characterizing Term Order Filtration	510
	52.4	Subalgebra Bases	514
	52.5	SAGBI Bases for Polynomial Invariants of Alternating Groups	519
	52.6	Finiteness of SAGBI Bases	528
	52.7	Göbel's Basis for Invariant Rings	531
	52.8	Buchberger's Algorithm for Symideals	536
	52.9	Aschenbrenner–Hillar: Buchberger Theory for Symmetric Ideals	542
	52.10	Variations on SAGBI	552
53	**Hironaka**		565
	53.1	A Theory of Normal Forms	567
	53.2	Normal Forms and Standard Bases	572
	53.3	The Zariskification of the Polynomial Ring	576
	53.4	The Tangent Cone Algorithm	581
	53.5	Standard Basis Algorithms	587
	53.6	The Normal Form Algorithm	589
	53.7	L-adic Valuation, Localization, Zariskification	592
	53.8	Standard Bases in Local Rings	593
54	**Hironaka II**		597
	54.1	Alonso–Luengo–Raimondo Model for Algebraic Power Series	597
	54.2	Canonical Forms in the Ring of Algebraic Power Series	601
	54.3	Standard Bases in the Ring of Algebraic Power Series	604
	54.4	An Effective Version of the Weierstrass Preparation Theorem	611

54.5 Alonso–Castro–Hauser: An Effective Version of Hironaka's Henselian Weierstrass Theorem 620
54.6 Branches of a Curve at a Singular Point 622
54.7 Toward Mac Millan's Algorithm 626
54.8 Mac Millan's Algorithm 630

55 Janet 633
55.1 Janet Decomposition 635
55.2 Riquier–Janet Completion 651
55.3 Degree-compatible Term Ordering and Convergency 658
55.4 Janet and Hilbert 661
55.5 Janet: Involutiveness 664
55.6 Term Orderings and the Borel Property in Riquier and His Followers 682

56 Macaulay V 691
56.1 Macaulay's Formula 692
56.2 Hilbert Function 700
56.3 Sperner's Proof of Macaulay's Formula 702
56.4 Gunther: Borel Relation and Borel Invariancy of Generic Forms 708
56.5 Gunther: Macaulay-like Formula for the Borel Set 716
56.6 Gunther's Proof of Macaulay's Formula 720
56.7 Gunther: Hilbert Function Computation 729
56.8 Gunther: Riquier–Janet Procedure 734
56.9 Bigatti–Hulett: Betti Numbers and Lex Segments 740
56.10 Marinari–Ramella: Revlex Segments 744
56.11 Cioffi–Roggero: J-marked Families 750

57 Gerdt and Faugère 757
57.1 Gerdt 758
57.2 Faugère 778
57.3 Mayr: Space-efficient Gröbner Basis Computation 796
57.4 Congedo: Buchberger–Wiesinger-Widi Groebnerian-Gaussian Bound 800

Bibliography 803
Index 813