

# Contents

<b>1</b>	<b>What's Quantum Optics?</b>	<b>1</b>
1.1	On the Road to Quantum Optics . . . . .	1
1.2	Resonance Fluorescence . . . . .	2
1.2.1	Elastic Peak: Light as a Wave . . . . .	2
1.2.2	Mollow-Three-Peak Spectrum . . . . .	3
1.2.3	Anti-Bunching . . . . .	5
1.3	Squeezing the Fluctuations . . . . .	7
1.3.1	What is a Squeezed State? . . . . .	7
1.3.2	Squeezed States in the Optical Parametric Oscillator . . . . .	9
1.3.3	Oscillatory Photon Statistics . . . . .	12
1.3.4	Interference in Phase Space . . . . .	13
1.4	Jaynes-Cummings-Paul Model . . . . .	14
1.4.1	Single Two-Level Atom plus a Single Mode . . . . .	15
1.4.2	Time Scales . . . . .	15
1.5	Cavity QED . . . . .	16
1.5.1	An Amazing Maser . . . . .	16
1.5.2	Cavity QED in the Optical Domain . . . . .	19
1.6	de Broglie Optics . . . . .	22
1.6.1	Electron and Neutron Optics . . . . .	22
1.6.2	Atom Optics . . . . .	23
1.6.3	Atom Optics in Quantized Light Fields . . . . .	25
1.7	Quantum Motion in Paul Traps . . . . .	26
1.7.1	Analogy to Cavity QED . . . . .	26
1.7.2	Quantum Information Processing . . . . .	26
1.8	Two-Photon Interferometry and More . . . . .	28
1.9	Outline of the Book . . . . .	29
<b>2</b>	<b>Ante</b>	<b>35</b>
2.1	Position and Momentum Eigenstates . . . . .	36
2.1.1	Properties of Eigenstates . . . . .	36
2.1.2	Derivative of Wave Function . . . . .	38
2.1.3	Fourier Transform Connects $x$ - and $p$ -Space . . . . .	39
2.2	Energy Eigenstate . . . . .	40
2.2.1	Arbitrary Representation . . . . .	41
2.2.2	Position Representation . . . . .	42
2.3	Density Operator: A Brief Introduction . . . . .	44

2.3.1	A State Vector is not Enough!	44
2.3.2	Definition and Properties	48
2.3.3	Trace of Operator	49
2.3.4	Examples of a Density Operator	51
2.4	Time Evolution of Quantum States	53
2.4.1	Motion of a Wave Packet	54
2.4.2	Time Evolution due to Interaction	55
2.4.3	Time Dependent Hamiltonian	57
2.4.4	Time Evolution of Density Operator	61
<b>3</b>	<b>Wigner Function</b>	<b>67</b>
3.1	Jump Start of the Wigner Function	68
3.2	Properties of the Wigner Function	69
3.2.1	Marginals	69
3.2.2	Overlap of Quantum States as Overlap in Phase Space	71
3.2.3	Shape of Wigner Function	72
3.3	Time Evolution of Wigner Function	74
3.3.1	von Neumann Equation in Phase Space	74
3.3.2	Quantum Liouville Equation	75
3.4	Wigner Function Determined by Phase Space	76
3.4.1	Definition of Moyal Function	76
3.4.2	Phase Space Equations for Moyal Functions	77
3.5	Phase Space Equations for Energy Eigenstates	78
3.5.1	Power Expansion in Planck's Constant	79
3.5.2	Model Differential Equation	81
3.6	Harmonic Oscillator	84
3.6.1	Wigner Function as Wave Function	85
3.6.2	Phase Space Enforces Energy Quantization	86
3.7	Evaluation of Quantum Mechanical Averages	87
3.7.1	Operator Ordering	88
3.7.2	Examples of Weyl-Wigner Ordering	90
<b>4</b>	<b>Quantum States in Phase Space</b>	<b>99</b>
4.1	Energy Eigenstate	100
4.1.1	Simple Phase Space Representation	100
4.1.2	Large- $m$ Limit	101
4.1.3	Wigner Function	105
4.2	Coherent State	108
4.2.1	Definition of a Coherent State	109
4.2.2	Energy Distribution	110
4.2.3	Time Evolution	113
4.3	Squeezed State	119
4.3.1	Definition of a Squeezed State	121
4.3.2	Energy Distribution: Exact Treatment	125
4.3.3	Energy Distribution: Asymptotic Treatment	128
4.3.4	Limit Towards Squeezed Vacuum	132

4.3.5	Time Evolution . . . . .	135
4.4	Rotated Quadrature States . . . . .	136
4.4.1	Wigner Function of Position and Momentum States . . . . .	137
4.4.2	Position Wave Function of Rotated Quadrature States . . . . .	140
4.4.3	Wigner Function of Rotated Quadrature States . . . . .	142
4.5	Quantum State Reconstruction . . . . .	143
4.5.1	Tomographic Cuts through Wigner Function . . . . .	143
4.5.2	Radon Transformation . . . . .	144
<b>5</b>	<b>Waves à la WKB</b>	<b>153</b>
5.1	Probability for Classical Motion . . . . .	153
5.2	Probability Amplitudes for Quantum Motion . . . . .	155
5.2.1	An Educated Guess . . . . .	156
5.2.2	Range of Validity of WKB Wave Function . . . . .	158
5.3	Energy Quantization . . . . .	159
5.3.1	Determining the Phase . . . . .	159
5.3.2	Bohr-Sommerfeld-Kramers Quantization . . . . .	161
5.4	Summary . . . . .	163
5.4.1	Construction of Primitive WKB Wave Function . . . . .	163
5.4.2	Uniform Asymptotic Expansion . . . . .	164
<b>6</b>	<b>WKB and Berry Phase</b>	<b>171</b>
6.1	Berry Phase and Adiabatic Approximation . . . . .	172
6.1.1	Adiabatic Theorem . . . . .	172
6.1.2	Analysis of Geometrical Phase . . . . .	174
6.1.3	Geometrical Phase as a Flux in Hilbert Space . . . . .	175
6.2	WKB Wave Functions from Adiabaticity . . . . .	176
6.2.1	Energy Eigenvalue Problem as Propagation Problem . . . . .	177
6.2.2	Dynamical and Geometrical Phase . . . . .	181
6.2.3	WKB Waves Rederived . . . . .	183
6.3	Non-Adiabatic Berry Phase . . . . .	185
6.3.1	Derivation of the Aharonov-Anandan Phase . . . . .	186
6.3.2	Time Evolution in Harmonic Oscillator . . . . .	187
<b>7</b>	<b>Interference in Phase space</b>	<b>189</b>
7.1	Outline of the Idea . . . . .	189
7.2	Derivation of Area-of-Overlap Formalism . . . . .	192
7.2.1	Jumps Viewed From Position Space . . . . .	192
7.2.2	Jumps Viewed From Phase Space . . . . .	197
7.3	Application to Franck-Condon Transitions . . . . .	200
7.4	Generalization . . . . .	201
<b>8</b>	<b>Applications of Interference in Phase Space</b>	<b>205</b>
8.1	Connection to Interference in Phase Space . . . . .	205
8.2	Energy Eigenstates . . . . .	206
8.3	Coherent State . . . . .	208
8.3.1	Elementary Approach . . . . .	209

	8.3.2	Influence of Internal Structure . . . . .	212
8.4		Squeezed State . . . . .	213
	8.4.1	Oscillations from Interference in Phase Space . . . . .	213
	8.4.2	Giant Oscillations . . . . .	216
	8.4.3	Summary . . . . .	218
8.5		The Question of Phase States . . . . .	221
	8.5.1	Amplitude and Phase in a Classical Oscillator . . . . .	221
	8.5.2	Definition of a Phase State . . . . .	223
	8.5.3	Phase Distribution of a Quantum State . . . . .	227
<b>9</b>		<b>Wave Packet Dynamics</b>	<b>233</b>
	9.1	What are Wave Packets? . . . . .	233
	9.2	Fractional and Full Revivals . . . . .	234
	9.3	Natural Time Scales . . . . .	237
	9.3.1	Hierarchy of Time Scales . . . . .	237
	9.3.2	Generic Signal . . . . .	239
	9.4	New Representations of the Signal . . . . .	241
	9.4.1	The Early Stage of the Evolution . . . . .	241
	9.4.2	Intermediate Times . . . . .	244
	9.5	Fractional Revivals Made Simple . . . . .	246
	9.5.1	Gauss Sums . . . . .	246
	9.5.2	Shape Function . . . . .	246
<b>10</b>		<b>Field Quantization</b>	<b>255</b>
	10.1	Wave Equations for the Potentials . . . . .	256
	10.1.1	Derivation of the Wave Equations . . . . .	256
	10.1.2	Gauge Invariance of Electrodynamics . . . . .	257
	10.1.3	Solution of the Wave Equation . . . . .	260
	10.2	Mode Structure in a Box . . . . .	262
	10.2.1	Solutions of Helmholtz Equation . . . . .	262
	10.2.2	Polarization Vectors from Gauge Condition . . . . .	263
	10.2.3	Discreteness of Modes from Boundaries . . . . .	264
	10.2.4	Boundary Conditions on the Magnetic Field . . . . .	264
	10.2.5	Orthonormality of Mode Functions . . . . .	265
	10.3	The Field as a Set of Harmonic Oscillators . . . . .	266
	10.3.1	Energy in the Resonator . . . . .	267
	10.3.2	Quantization of the Radiation Field . . . . .	269
	10.4	The Casimir Effect . . . . .	272
	10.4.1	Zero-Point Energy of a Rectangular Resonator . . . . .	272
	10.4.2	Zero-Point Energy of Free Space . . . . .	274
	10.4.3	Difference of Two Infinite Energies . . . . .	275
	10.4.4	Casimir Force: Theory and Experiment . . . . .	276
	10.5	Operators of the Vector Potential and Fields . . . . .	278
	10.5.1	Vector Potential . . . . .	278
	10.5.2	Electric Field Operator . . . . .	280
	10.5.3	Magnetic Field Operator . . . . .	281

10.6	Number States of the Radiation Field . . . . .	281
10.6.1	Photons and Anti-Photons . . . . .	282
10.6.2	Multi-Mode Case . . . . .	282
10.6.3	Superposition and Entangled States . . . . .	283
<b>11</b>	<b>Field States</b>	<b>291</b>
11.1	Properties of the Quantized Electric Field . . . . .	291
11.1.1	Photon Number States . . . . .	292
11.1.2	Electromagnetic Field Eigenstates . . . . .	293
11.2	Coherent States Revisited . . . . .	295
11.2.1	Eigenvalue Equation . . . . .	295
11.2.2	Coherent State as a Displaced Vacuum . . . . .	297
11.2.3	Photon Statistics of a Coherent State . . . . .	298
11.2.4	Electric Field Distribution of a Coherent State . . . . .	299
11.2.5	Over-completeness of Coherent States . . . . .	301
11.2.6	Expansion into Coherent States . . . . .	303
11.2.7	Electric Field Expectation Values . . . . .	305
11.3	Schrödinger Cat State . . . . .	306
11.3.1	The Original Cat Paradox . . . . .	306
11.3.2	Definition of the Field Cat State . . . . .	307
11.3.3	Wigner Phase Space Representation . . . . .	307
11.3.4	Photon Statistics . . . . .	310
<b>12</b>	<b>Phase Space Functions</b>	<b>321</b>
12.1	There is more than Wigner Phase Space . . . . .	321
12.1.1	Who Needs Phase Space Functions? . . . . .	321
12.1.2	Another Description of Phase Space . . . . .	322
12.2	The Husimi-Kano $Q$ -Function . . . . .	324
12.2.1	Definition of $Q$ -Function . . . . .	324
12.2.2	$Q$ -Functions of Specific Quantum States . . . . .	324
12.3	Averages Using Phase Space Functions . . . . .	330
12.3.1	Heuristic Argument . . . . .	330
12.3.2	Rigorous Treatment . . . . .	333
12.4	The Glauber-Sudarshan $P$ -Distribution . . . . .	337
12.4.1	Definition of $P$ -Distribution . . . . .	337
12.4.2	Connection between $Q$ - and $P$ -Function . . . . .	338
12.4.3	$P$ -Function from $Q$ -Function . . . . .	339
12.4.4	Examples of $P$ -Distributions . . . . .	341
<b>13</b>	<b>Optical Interferometry</b>	<b>349</b>
13.1	Beam Splitter . . . . .	350
13.1.1	Classical Treatment . . . . .	350
13.1.2	Symmetric Beam Splitter . . . . .	352
13.1.3	Transition to Quantum Mechanics . . . . .	353
13.1.4	Transformation of Quantum States . . . . .	353
13.1.5	Count Statistics at the Exit Ports . . . . .	356
13.2	Homodyne Detector . . . . .	357

13.2.1	Classical Considerations . . . . .	357
13.2.2	Quantum Treatment . . . . .	358
13.3	Eight-Port Interferometer . . . . .	361
13.3.1	Quantum State of the Output Modes . . . . .	361
13.3.2	Photon Count Statistics . . . . .	363
13.3.3	Simultaneous Measurement and EPR . . . . .	365
13.3.4	Q-Function Measurement . . . . .	367
13.4	Measured Phase Operators . . . . .	370
13.4.1	Measurement of Classical Trigonometry . . . . .	370
13.4.2	Measurement of Quantum Trigonometry . . . . .	372
13.4.3	Two-Mode Phase Operators . . . . .	374
<b>14</b>	<b>Atom-Field Interaction</b> . . . . .	<b>381</b>
14.1	How to Construct the Interaction? . . . . .	382
14.2	Vector Potential-Momentum Coupling . . . . .	382
14.2.1	Gauge Principle Determines Minimal Coupling . . . . .	383
14.2.2	Interaction of an Atom with a Field . . . . .	386
14.3	Dipole Approximation . . . . .	389
14.3.1	Expansion of Vector Potential . . . . .	389
14.3.2	$\vec{A} \cdot \vec{p}$ -Interaction . . . . .	390
14.3.3	Various Forms of the $\vec{A} \cdot \vec{p}$ Interaction . . . . .	390
14.3.4	Higher Order Corrections . . . . .	392
14.4	Electric Field-Dipole Interaction . . . . .	393
14.4.1	Dipole Approximation . . . . .	393
14.4.2	Röntgen Hamiltonians and Others . . . . .	393
14.5	Subsystems, Interaction and Entanglement . . . . .	395
14.6	Equivalence of $\vec{A} \cdot \vec{p}$ and $\vec{r} \cdot \vec{E}$ . . . . .	396
14.6.1	Classical Transformation of Lagrangian . . . . .	397
14.6.2	Quantum Mechanical Treatment . . . . .	399
14.6.3	Matrix elements of $\vec{A} \cdot \vec{p}$ and $\vec{r} \cdot \vec{E}$ . . . . .	399
14.7	Equivalence of Hamiltonians $H^{(1)}$ and $\tilde{H}^{(1)}$ . . . . .	400
14.8	Simple Model for Atom-Field Interaction . . . . .	402
14.8.1	Derivation of the Hamiltonian . . . . .	402
14.8.2	Rotating-Wave Approximation . . . . .	406
<b>15</b>	<b>Jaynes-Cummings-Paul Model: Dynamics</b> . . . . .	<b>413</b>
15.1	Resonant Jaynes-Cummings-Paul Model . . . . .	413
15.1.1	Time Evolution Operator Using Operator Algebra . . . . .	414
15.1.2	Interpretation of Time Evolution Operator . . . . .	416
15.1.3	State Vector of Combined System . . . . .	418
15.1.4	Dynamics Represented in State Space . . . . .	418
15.2	Role of Detuning . . . . .	420
15.2.1	Atomic and Field States . . . . .	420
15.2.2	Rabi Equations . . . . .	422
15.3	Solution of Rabi Equations . . . . .	423
15.3.1	Laplace Transformation . . . . .	424

15.3.2	Inverse Laplace Transformation . . . . .	425
15.4	Discussion of Solution . . . . .	426
15.4.1	General Considerations . . . . .	427
15.4.2	Resonant Case . . . . .	427
15.4.3	Far Off-Resonant Case . . . . .	429
<b>16 State Preparation and Entanglement</b>		<b>435</b>
16.1	Measurements on Entangled Systems . . . . .	435
16.1.1	How to Get Probabilities . . . . .	436
16.1.2	State of the Subsystem after a Measurement . . . . .	439
16.1.3	Experimental Setup . . . . .	440
16.2	Collapse, Revivals and Fractional Revivals . . . . .	444
16.2.1	Inversion as Tool for Measuring Internal Dynamics . . . . .	444
16.2.2	Experiments on Collapse and Revivals . . . . .	447
16.3	Quantum State Preparation . . . . .	451
16.3.1	State Preparation with a Dispersive Interaction . . . . .	451
16.3.2	Generation of Schrödinger Cats . . . . .	454
16.4	Quantum State Engineering . . . . .	454
16.4.1	Outline of the Method . . . . .	454
16.4.2	Inverse Problem . . . . .	458
16.4.3	Example: Preparation of a Phase State . . . . .	461
<b>17 Paul Trap</b>		<b>473</b>
17.1	Basics of Trapping Ions . . . . .	474
17.1.1	No Static Trapping in Three Dimensions . . . . .	474
17.1.2	Dynamical Trapping . . . . .	475
17.2	Laser Cooling . . . . .	479
17.3	Motion of an Ion in a Paul Trap . . . . .	480
17.3.1	Reduction to Classical Problem . . . . .	481
17.3.2	Motion as a Sequence of Squeezing and Rotations . . . . .	483
17.3.3	Dynamics in Wigner Phase Space . . . . .	486
17.3.4	Floquet Solution . . . . .	490
17.4	Model Hamiltonian . . . . .	494
17.4.1	Transformation to Interaction Picture . . . . .	495
17.4.2	Lamb-Dicke Regime . . . . .	496
17.4.3	Multi-Phonon Jaynes-Cummings-Paul Model . . . . .	498
17.5	Effective Potential Approximation . . . . .	500
<b>18 Damping and Amplification</b>		<b>507</b>
18.1	Damping and Amplification of a Cavity Field . . . . .	508
18.2	Density Operator of a Subsystem . . . . .	509
18.2.1	Coarse-Grained Equation of Motion . . . . .	509
18.2.2	Time Independent Hamiltonian . . . . .	511
18.3	Reservoir of Two-Level Atoms . . . . .	511
18.3.1	Approximate Treatment . . . . .	512
18.3.2	Density Operator in Number Representation . . . . .	514
18.3.3	Exact Master Equation . . . . .	519

18.3.4	Summary . . . . .	522
18.4	One-Atom Maser . . . . .	523
18.4.1	Density Operator Equation . . . . .	524
18.4.2	Equation of Motion for the Photon Statistics . . . . .	529
18.4.3	Phase Diffusion . . . . .	532
18.5	Atom-Reservoir Interaction . . . . .	532
18.5.1	Model and Equation of Motion . . . . .	533
18.5.2	First Order Contribution . . . . .	535
18.5.3	Bloch Equations . . . . .	537
18.5.4	Second Order Contribution . . . . .	539
18.5.5	Lamb Shift . . . . .	540
18.5.6	Weisskopf-Wigner Decay . . . . .	540
<b>19</b>	<b>Atom Optics in Quantized Light Fields</b>	<b>549</b>
19.1	Formulation of Problem . . . . .	549
19.1.1	Dynamics . . . . .	549
19.1.2	Time Evolution of Probability Amplitudes . . . . .	552
19.2	Reduction to One-Dimensional Scattering . . . . .	554
19.2.1	Slowly Varying Approximation . . . . .	554
19.2.2	From Two Dimensions to One . . . . .	555
19.2.3	State Vector . . . . .	556
19.3	Raman-Nath Approximation . . . . .	557
19.3.1	Heuristic Arguments . . . . .	557
19.3.2	Probability Amplitudes . . . . .	558
19.4	Deflection of Atoms . . . . .	559
19.4.1	Measurement Schemes and Scattering Conditions . . . . .	559
19.4.2	Kapitza-Dirac Regime . . . . .	562
19.4.3	Kapitza-Dirac Scattering with a Mask . . . . .	568
19.5	Interference in Phase Space . . . . .	571
19.5.1	How to Represent the Quantum State? . . . . .	572
19.5.2	Area of Overlap . . . . .	572
19.5.3	Expression for Probability Amplitude . . . . .	573
<b>20</b>	<b>Wigner Functions in Atom Optics</b>	<b>579</b>
20.1	Model . . . . .	579
20.2	Equation of Motion for Wigner Functions . . . . .	581
20.3	Motion in Phase Space . . . . .	582
20.3.1	Harmonic Approximation . . . . .	583
20.3.2	Motion of the Atom in the Cavity . . . . .	583
20.3.3	Motion of the Atom outside the Cavity . . . . .	585
20.3.4	Snap Shots of the Wigner Function . . . . .	586
20.4	Quantum Lens . . . . .	587
20.4.1	Distributions of Atoms in Space . . . . .	587
20.4.2	Focal Length and Deflection Angle . . . . .	589
20.5	Photon and Momentum Statistics . . . . .	590
20.6	Heuristic Approach . . . . .	592



20.6.1	Focal Length . . . . .	592
20.6.2	Focal Size . . . . .	594
<b>A Energy Wave Functions of Harmonic Oscillator</b>		<b>597</b>
A.1	Polynomial Ansatz . . . . .	597
A.2	Asymptotic Behavior . . . . .	599
A.2.1	Energy Wave Function as a Contour Integral . . . . .	600
A.2.2	Evaluation of the Integral $I_m$ . . . . .	600
A.2.3	Asymptotic Limit of $f_m$ . . . . .	603
A.2.4	Bohr's Correspondence Principle . . . . .	603
<b>B Time Dependent Operators</b>		<b>605</b>
B.1	Caution when Differentiating Operators . . . . .	605
B.2	Time Ordering . . . . .	606
B.2.1	Product of Two Terms . . . . .	607
B.2.2	Product of $n$ Terms . . . . .	608
<b>C Süßmann Measure</b>		<b>611</b>
C.1	Why Other Measures Fail . . . . .	611
C.2	One Way out of the Problem . . . . .	612
C.3	Generalization to Higher Dimensions . . . . .	613
<b>D Phase Space Equations</b>		<b>615</b>
D.1	Formulation of the Problem . . . . .	615
D.2	Fourier Transform of Matrix Elements . . . . .	616
D.3	Kinetic Energy Terms . . . . .	617
D.4	Potential Energy Terms . . . . .	619
D.5	Summary . . . . .	620
<b>E Airy Function</b>		<b>621</b>
E.1	Definition and Differential Equation . . . . .	621
E.2	Asymptotic Expansion . . . . .	622
E.2.1	Oscillatory Regime . . . . .	623
E.2.2	Decaying Regime . . . . .	624
E.2.3	Stokes Phenomenon . . . . .	625
<b>F Radial Equation</b>		<b>629</b>
<b>G Asymptotics of a Poissonian</b>		<b>633</b>
<b>H Toolbox for Integrals</b>		<b>635</b>
H.1	Method of Stationary Phase . . . . .	635
H.1.1	One-Dimensional Integrals . . . . .	635
H.1.2	Multi-Dimensional Integrals . . . . .	637
H.2	Cornu Spiral . . . . .	639

	<b>643</b>
<b>I Area of Overlap</b>	<b>643</b>
I.1 Diamond Transformed into a Rectangle . . . . .	643
I.2 Area of Diamond . . . . .	644
I.3 Area of Overlap as Probability . . . . .	646
<b>J P-Distributions</b>	<b>649</b>
J.1 Thermal State . . . . .	649
J.2 Photon Number State . . . . .	650
J.3 Squeezed State . . . . .	651
<b>K Homodyne Kernel</b>	<b>655</b>
K.1 Explicit Evaluation of Kernel . . . . .	655
K.2 Strong Local Oscillator Limit . . . . .	656
<b>L Beyond the Dipole Approximation</b>	<b>659</b>
L.1 First Order Taylor Expansion . . . . .	659
L.1.1 Expansion of the Hamiltonian . . . . .	659
L.1.2 Extension to Operators . . . . .	661
L.2 Classical Gauge Transformation . . . . .	661
L.2.1 Lagrangian with Center-of-Mass Motion . . . . .	662
L.2.2 Complete Time Derivative . . . . .	663
L.2.3 Hamiltonian Including Center-of-Mass Motion . . . . .	663
L.3 Quantum Mechanical Gauge Transformation . . . . .	664
L.3.1 Gauge Potential . . . . .	664
L.3.2 Schrödinger equation for $\tilde{\Phi}$ . . . . .	667
<b>M Effective Hamiltonian</b>	<b>669</b>
<b>N Oscillator Reservoir</b>	<b>671</b>
N.1 Second Order Contribution . . . . .	671
N.1.1 Evaluation of Double Commutator . . . . .	671
N.1.2 Trace over Reservoir . . . . .	673
N.2 Symmetry Relations in Trace . . . . .	673
N.2.1 Complex Conjugates . . . . .	674
N.2.2 Commutator Between Field Operators . . . . .	674
N.3 Master Equation . . . . .	675
N.4 Explicit Expressions for $\Gamma$ , $\beta$ and $\tilde{G}$ . . . . .	676
N.5 Integration over Time . . . . .	677
<b>O Bessel Functions</b>	<b>679</b>
O.1 Definition . . . . .	679
O.2 Asymptotic Expansion . . . . .	680
<b>P Square Root of <math>\delta</math></b>	<b>683</b>
<b>Q Further Reading</b>	<b>685</b>
Index	<b>688</b>