## Contents

Preface			
No	tes for the Reader	xi	
I	Aims and Scope of This Book	I	
2	Prehistory, Emergence, and Evolution of Fuzzy Logic         2.1       Prehistory of fuzzy logic         2.2       Emergence of fuzzy logic         2.3       Fuzzy logic	5 5 17	
	<ul> <li>2.3 Evolving attitudes toward ruzzy logic</li></ul>	27 32 38	
3	Fuzzy Logic in the Broad Sense3.1Introduction3.2Basic concepts of fuzzy sets3.3Operations on fuzzy sets3.4Fuzzy intervals, fuzzy numbers, and fuzzy arithmetic3.5Fuzzy relations3.6Approximate reasoning3.7Possibility theory3.8Fuzzy clustering3.9Methods for constructing fuzzy sets3.10Nonstandard fuzzy sets	43 43 44 46 55 60 69 79 88 94 99	
4	<ul> <li>Fuzzy Logic in the Narrow Sense</li> <li>4.1 From classical to fuzzy logic: Principal issues</li></ul>	105 106 112 112 129 134	

		4.2.4	Predicate logics			
		4.2.5	Further developments 155			
		4.2.6	The mid-1960s and Zadeh's idea of fuzzy sets 159			
	4.3	Fuzzy l	ogics with graded consequence			
		4.3.1	Goguen's logic of inexact concepts			
		4.3.2	Pavelka-style fuzzy logic			
		4.3.3	Further developments 171			
	4.4	Fuzzy l	ogics based on t-norms and their residua			
		4.4.I	Fuzzy logics based on t-norms until the mid-1990s 178			
		4.4.2	Product logic			
		4.4.3	Hájek's logic BL and <i>Metamathematics of Fuzzy Logic</i> 189			
		4.4.4	Logics related to BL 202			
	4.5	Fuzzy l	logic and uncertainty 210			
		4.5 <i>.</i> 1	Degrees of truth vs. belief and truth functionality 210			
		4.5.2	Possibilistic logic			
		4.5.3	Gerla's probabilistic fuzzy logics 215			
		4.5.4	Belief, modality, and quantifiers in fuzzy logic 216			
	4.6	Miscel	laneous issues			
		4.6.1	Relationship to applications			
		4.6.2	Computability and complexity 224			
		4.6.3	Further developments 227			
	3.7					
5	Mat	Mathematics Based on Fuzzy Logic 231				
	5.1	Princij	pal issues and outline of development 231			
		5.1.1	What is mathematics based on fuzzy logic? 231			
		5.1.2	The problem and role of foundations			
		5.1.3	The problem and role of applications			
		5.1.4	Outline of development			
	5.2	Found	lations of mathematics based on fuzzy logic			
		5.2.1	I he role of fuzzy logic in the narrow sense			
		5.2.2	Higher-order logic approaches 250			
		5.2.3	Set-theoretic approaches 253			
		5.2.4	Category-theoretic approaches			
	5.3	Selecte	ed areas of mathematics based on fuzzy logic			
		5.3.I	Sets and relations 267			
		5.3.2	Algebra			
		6 2 2	LODOLOGY			
		3.3.3				
		5.3.4 5.3.4	Quantities and mathematical analysis			
		5.3.4 5.3.5	Quantities and mathematical analysis       298         Probability and statistics       309			
		5.3.4 5.3.5 5.3.6	Quantities and mathematical analysis       298         Probability and statistics       309         Geometry       315			

## Contents

	5.4	Miscel 5.4.1 5.4.2 5.4.3	aneous issues321Interpretation of truth degrees321Fuzzy logic and paradoxes332Fuzzy logic and vagueness339					
6	Appl	ications	of Fuzzy Logic 347					
	6.1	Introd	uction					
	6.2	A histo	A historical overview 349					
	6.3	Engine	ering					
		6.3.1	Fuzzy control					
		6.3.2	Other engineering applications 365					
	6.4	Decision making						
	6.5	Natura	al sciences					
		6.5.1	Chemistry					
		6.5.2	Biology 376					
		6.5.3	Physics					
	6.6	Earth s	ciences					
	6.7	Psychology						
	6.8	Social	sciences					
		6.8.1	A historical overview					
		6.8.2	Economics					
		6.8.3	Political science					
	6.9	Comp	uter science					
	6.10	Medic	ine					
	6.11	Manag	gement and business					
	6.12	Other	applications					
7	Signi	ficance	of Fuzzy Logic 421					
<i>'</i>	7.1	Introd	uction					
	7.2	A retro	ospective overview of fuzzy logic					
	7.3	Paradigm shifts in science, mathematics, engineering, and other areas 425						
	7.4	Assess	ment of the significance of fuzzy logic					
	7.5	A pros	spective view on the soth anniversary of fuzzy logic					
	1.5	F - 54						

## Appendices

A	The Enigma of Cox's Proof	449
B	Overview of Classical Logic	453
С	Photographs	469

Glossary of Symbols	473
References	475
Name Index	511
Subject Index	521