Contents

1 Scalar Reaction-Diffusion Equations: Conditional Symmetry, Exact Solutions and Applications 1

1.1 Nonlinear Reaction-Diffusion Equations in Mathematical Biology 1

1.2 Nonclassical Symmetry: Historical Review, Definitions and Properties 4

1.3 Examples of Q-Conditional Symmetries of Some Reaction-Diffusion Equations 9

1.4 Determining Equations for the General Reaction-Diffusion-Convection Equation 13

1.5 Q-Conditional Symmetry of Reaction-Diffusion-Convection Equations with Constant Diffusivity 17

1.6 Exact Solutions of Some Equations Arising in Biological Models ... 21

1.7 Q-Conditional Symmetry of Reaction-Diffusion-Convection Equations with Variable Diffusivity 33

1.8 Exact Solutions of Some Equations with Power-Law Diffusivity and Their Interpretation 36

1.9 Concluding Remarks 40

References 41

2 Q-Conditional Symmetries of Reaction-Diffusion Systems 45

2.1 Reaction-Diffusion Systems and Their Applications 45

2.2 Q-Conditional Symmetry for Systems of Partial Differential Equations 48

2.3 Systems of Determining Equations 52

2.4 Conditional Symmetries of Reaction-Diffusion Systems with Constant Diffusivities 56

2.5 Conditional Symmetries of Reaction-Diffusion Systems with Power-Law Diffusivities 61

2.6 Concluding Remarks 72

References 73

3 Conditional Symmetries and Exact Solutions of Diffusive Lotka–Volterra Systems 77
3.1 The Lotka–Volterra System and Its Application 77
3.2 The Two-Component Diffusive Lotka–Volterra System 79
3.2.1 Determining Equations 79
3.2.2 Q-Conditional Symmetry 82
3.2.3 Reductions to Systems of Ordinary Differential Equations and Exact Solutions 89
3.3 The Three-Component Diffusive Lotka–Volterra System 98
3.3.1 Lie Symmetry 99
3.3.2 Determining Equations 101
3.3.3 Q-Conditional Symmetry of the First Type 102
3.3.4 Exact Solutions and Their Interpretation 107
3.4 A Hunter-Gatherer–Farmer Population Model 112
3.5 Concluding Remarks 115
References 117

4 Q-Conditional Symmetries of the First Type and Exact Solutions of Nonlinear Reaction-Diffusion Systems 119
4.1 Determining Equations 119
4.2 Reaction-Diffusion Systems with Constant Diffusivities 122
4.2.1 Q-Conditional Symmetry of the First Type 123
4.2.2 Reductions, Exact Solutions and Their Interpretation 132
4.3 Reaction-Diffusion Systems with Variable Diffusivities 137
4.3.1 Q-Conditional Symmetry of the First Type 137
4.3.2 Reductions and Exact Solutions 143
4.3.3 Application to a Physically Motivated Problem 145
4.4 Concluding Remarks 147
References 151

A List of Reaction-Diffusion Systems and Exact Solutions 155

Index 157