## Contents

| 1 | Representation and Knowledge: The Semiotic Revolution     |  |   |     |  |  |  |  |  |
|---|---|--|---|-----|--|--|--|--|--|
|   | 1.1   | The Fundamental Epistemological Distinction  |   |     |  |  |  |  |  |
|   |   | and th   | e First Knowledge Analysis Scheme                         | 2   |  |  |  |  |  |
|   |   | 1.1.1  | The Cognitive Issue of Access to the Objects Themselves   |     |  |  |  |  |  |
|   |   |  | and Role of Representations                               | 4   |  |  |  |  |  |
|   |   | 1.1.2  | Sign and Representation: The Cognitive Divide             | 5   |  |  |  |  |  |
|   | 1.2   | The S  | emiotic Revolution: Towards a New Knowledge Analysis      |     |  |  |  |  |  |
|   | Scheme  |  | ne  | 8   |  |  |  |  |  |
|   | 1.3   | The Three Models of Sign Analysis That Have Founded  |   |     |  |  |  |  |  |
|   |   | Semiotics: Contributions and Limits  |   |     |  |  |  |  |  |
|   |   | 1.3.1  | Saussure: Structural Analysis of Semiotic Systems         | 12  |  |  |  |  |  |
|   |   | 1.3.2  | Peirce: The Classification of Kinds of Representations    | 14  |  |  |  |  |  |
|   |   | 1.3.3  | Frege: Semiotic Substitution and Production of New        |     |  |  |  |  |  |
|   |   |  | Knowledge in Mathematics                                  | 15  |  |  |  |  |  |
|   | 1.4   | Concl  | usion: The Semiotic Representations                       | 17  |  |  |  |  |  |
|   | Ann   | Annex  |   |     |  |  |  |  |  |
| 2 | Mathematical Activity and the Transformations of Semiotic |  |   |     |  |  |  |  |  |
| - | Depresentations   |  |   |     |  |  |  |  |  |
|   | 2 1   | initial structure of the structure of th | 21  |     |  |  |  |  |  |
|   | <i>4</i> .1   | in the Access to Objects of Knowledge  |   |     |  |  |  |  |  |
|   |   | 11 ute   | The Instance ition Test with a Material Objects The Deate | 25  |  |  |  |  |  |
|   |   | 2.1.1  | Montoso of Vosuth   | 22  |  |  |  |  |  |
|   |   | 010  | The Least on a sitil and with the Network Newsham         | 23  |  |  |  |  |  |
|   |   | 2.1.2  | The Juxtaposition fest with the Natural Numbers           | 25  |  |  |  |  |  |
|   |   | 2.1.3  | How to Recognize the Same Object in Different             |     |  |  |  |  |  |
|   |   |  | Representations?  | 27  |  |  |  |  |  |
|   |   | 2.1.4  | A Fundamental Cognitive Operation in Mathematics:         | • • |  |  |  |  |  |
|   | • •   |  | One-to-One Mapping  | 29  |  |  |  |  |  |
|   | 2.2   | 2.2 The Transformation of Semiotic Representations at the Heart  |   |     |  |  |  |  |  |
|   |   | of the Mathematical Way of Working   |   |     |  |  |  |  |  |

|   |      | 2.2.1      | Description of an Elementary Mathematical Activity:<br>The Development of Polygonal Unit Marks |          |
|---|------|------------|--|----------|
|   |      |            | Configuration  | 32       |
|   |      | 2.2.2      | Representational Transformations Specific to each Kind   |          |
|   |      |            | of Semiotic Representation: The Case of Representation   |          |
|   |      |            | of Numbers   | 36       |
|   | 2.3  | Conclu     | usion: The Cognitive Analysis of the Mathematical Activity                                     |          |
|   |      | and the    | e Functioning of the Mathematical Thinking   | 41       |
| 2 | Rog  | sters of   | Semiotic Representations and Analysis of the Cognitive   |          |
| 5 | Fun  | ctioning   | a of Mathematical Thinking   | 45       |
|   | 2 1  | Semio      | tic Registers and Cognitive Functioning of Thought   | 47       |
|   | 5.1  | 3 1 1      | Two Heterogeneous Kinds of Semiotic Systems:   |          |
|   |      | 5.1.1      | The Codes and Registers  | 47       |
|   |      | 212        | The Three Types of Discursive Operations   |          |
|   |      | 3.1.2      | and the Cognitive Functions of Natural Languages   | 51       |
|   |      | 212        | The Palationship Batween Thought and Languages   | 51       |
|   |      | 5.1.5      | Discursive Operations and Linguistic Expression  | 54       |
|   |      | 214        | Conclusion: What Characterizes a Degister of Semiotic  | 54       |
|   |      | 5.1.4      | Conclusion: what Characterizes a Register of Sennouc   | 56       |
|   | 2.2  | Dath       | Representation.  | 50       |
|   | 3.2  | Do the     | d on Depicture?  | 57       |
|   |      | Depen      | d on Registers?  | 59       |
|   |      | 3.2.1      | How do we see a Figure 1   | 50       |
|   |      | 3.2.2      | the Two Types of Figural Operations Proper   | 61       |
|   |      | 202        | Concentration of the Desister of Elements in the Teaching                                      | 01       |
|   |      | 3.2.3      | Concealment of the Register of Figures in the Teaching   | 62       |
|   |      | 224        | of Geometry and Didactic Analyses  | 05       |
|   |      | 3.2.4      | Geometric Visualization and Problems from Reality: Direct                                      | 65       |
|   | 2.2  | <b>a</b> 1 | Passage or Need for Intermediary Representations?  | 63<br>67 |
|   | 3.3  | Conci      | usions   | 07       |
| 4 | The  | Registe    | ers: Method of Analysis and Identification   |          |
|   | of C | ognitiv    | e Variables  | 73       |
|   | 4.1  | How t      | o Isolate and Recognize the Meaning Units Mathematically                                       |          |
|   |      | Releva     | ant in the Content of a Representation?  | 75       |
|   |      | 4.1.1      | Production of Graphs and Their Equivocal   |          |
|   |      |            | Obviousness  | 76       |
|   |      | 4.1.2      | Methodology to Isolate the Mathematically Relevant   |          |
|   |      |            | Meaning Units in Any Representation Content  | 77       |
|   |      | 4.1.3      | What Kind of Task for Developing the Recognition   |          |
|   |      |            | of Mathematically Relevant Meaning Units?  | 81       |
|   | 4.2  | The A      | nalysis of Mathematical Activity Based on the Couples  |          |
|   |      | of Mo      | bilized Registers  | 83       |
|   |      | 4.2.1      | Congruence and Non-Congruence Phenomena  |          |
|   |      |            | in the Conversion of the Representations   | 86       |
|   |      |            |  |          |

|        | 4.2.2  | The Particular Role of Natural Language in the        |     |
|--------|--|---|-----|
|        |  | Cognitive Functioning Subjacent to the Mathematical   |     |
|        | 400  | Reasoning.  | 90  |
|        | 4.2.3  | The Understanding of the Problem Statements and the   |     |
|        | 40.4   | Need for Transitional Auxiliary Representations       | 91  |
|        | 4.2.4  | The Problem of Cognitive Connection Between           |     |
|        |  | the Natural Language and Other Registers              | 95  |
| 4.3    | Functional Variations of Phenomenological Production Methods |   |     |
|        | and Se   | emiotic Representation Registers                      | 96  |
|        | 4.3.1  | The Misleading Confusion Between Functional           |     |
|        |  | and Structural Variations in Production               |     |
|        |  | of Representations                                    | 97  |
|        | 4.3.2  | The Computer Monitors: Another Phenomenological       |     |
|        |  | Mode of Production of Representations                 | 99  |
| 4.4    | Metho  | od of Analysis of the Activities in Class and Student |     |
|        | Productions: The Problem of Didactically Relevant Variables  |   |     |
|        | 4.4.1  | The Organization of Sequences of Activities Always    |     |
|        |  | Has Two Sides   | 102 |
|        | 4.4.2  | The Field of Work Cognitively Required for a Teaching |     |
|        |  | Sequence of Geometrical Activities at Primary School  | 104 |
|        | 4.4.3  | Observation of the Students and Analysis of Their     |     |
|        |  | Productions and Reactions                             | 105 |
|        | 4.4.4  | Interactions and Cognitive Impact of Three Kinds      |     |
|        |  | of Verbalization on Understanding                     | 107 |
| 45     | Conch  |   | 108 |
|        | Contra   |   |     |
| Annex. |  |   | 113 |
| Anal   | vsis of  | an Example of Introduction of the Linear Function     |     |
| Conc   | ept in a   | a Textbook for Students Aged 13–14 Years Old          | 113 |
| 2011   | -P   |   |     |
| Index  |  |   | 115 |