

# Contents

<b>1 Introduction</b> .....	1
1.1 Contents of the Book .....	1
1.2 The Subject Field .....	2
1.3 The Development of the Subject Field Before the Last Millennium Shift .....	5
1.4 Recent Developments in Gravimetric Theory and Data .....	7
1.4.1 Development of Gravimetric Data.....	7
1.4.2 Development of Theory .....	8
1.5 Reference System, Reference Frame and Datum.....	10
1.5.1 More on Reference Systems .....	14
1.5.2 Different Types of Reference Systems.....	15
1.5.3 Major Geodynamical Effects on Reference Frames.....	17
1.5.4 Geodetic Reference System 1980 .....	22
References.....	23
<b>2 Basic Mathematics</b> .....	27
2.1 Least Squares Adjustment Theory .....	27
2.1.1 Adjustment by Elements .....	28
2.2 Least Squares Collocation.....	32
2.2.1 Discrete Collocation .....	32
2.2.2 Continuous Collocation.....	33
2.3 Coordinate Systems .....	35
2.4 Legendre's Polynomials .....	41
2.5 Spherical Harmonics.....	43
2.5.1 Spectral Filtering and Combination.....	46
2.6 Ellipsoidal Harmonics.....	56
2.7 Fundamentals of Potential Theory .....	57
2.7.1 Basic Concepts and Formulas .....	57
2.7.2 Laplace's and Poisson's Equations .....	60
2.7.3 Laplace's Equation and Its Solution in Spherical Coordinates.....	61

2.7.4	Gauss' and Green's Integral Formulas	62
2.7.5	Boundary Value Problems	65
2.8	Regularization	66
2.8.1	Tikhonov Regularization	69
2.8.2	Wiener Filtering	72
2.8.3	Spectral Smoothing	74
2.8.4	Spectral Combination	74
2.8.5	Optimum Regularization	76
2.8.6	Spherical Harmonic Analysis	78
2.8.7	Comparison	79
2.8.8	Concluding Remarks	80
	Appendix: Answers to Exercises	80
	References	81
<b>3</b>	<b>Classical Physical Geodesy</b>	<b>83</b>
3.1	Introduction	83
3.2	Basic Concepts in Physical Geodesy	84
3.2.1	The Gravity Field	84
3.2.2	The Gravity Field of the Level Ellipsoid	85
3.2.3	The Disturbing Potential, Geoid and Gravity Anomaly	89
3.2.4	Harmonic Expansion of the Gravity Field	92
3.3	Integral Formulas in Physical Geodesy	93
3.3.1	Poisson's Integral	94
3.3.2	Stokes' Formula	94
3.3.3	Hotine's Formula	96
3.3.4	Vening Meinesz' Integrals	97
3.3.5	The Vertical Gradient of Gravity	98
3.3.6	The Inverse Vening Meinesz Formula	99
3.3.7	The Geoid-from-Deflection Formula	101
3.3.8	Gradiometry Formulas on the Sphere	102
3.4	Practical Considerations (DITE, DWC, SITE, PITE)	106
3.4.1	The Free-Air Correction	106
3.4.2	The Bouguer Correction	107
3.4.3	The Direct Topographic Effect (DITE)	108
3.4.4	The SITE, Co-geoid and the PITE	108
3.5	Height Systems	110
3.5.1	Geopotential Numbers	110
3.5.2	Orthometric Heights	111
3.5.3	Normal Heights	112
3.5.4	Normal-Orthometric Heights	113
	Appendix 1: Closed-Form Kernels	115
	Appendix 2: Solutions to Exercises	116
	References	118

<b>4</b>	<b>Modern Physical Geodesy</b> .....	121
4.1	Introduction .....	121
4.2	The Quasigeoid, Surface Gravity Anomaly and Disturbance .....	126
4.3	Geoid Determination by Spherical Harmonics .....	128
4.4	The Modified Stokes' Formula .....	130
4.4.1	General Modification of Stokes' Formula .....	130
4.4.2	Remove-Restore Techniques .....	133
4.4.3	Modifications Reducing the Truncation Error .....	134
4.4.4	The Least Squares Modification of $N_1^{L,M}$ and $N_2^{L,M}$ .....	137
4.4.5	Satellite Only Low Degree Modifications .....	142
4.4.6	Modifications with High-Degree EGMs .....	144
4.5	Summary of Modified Stokes' Formula Techniques .....	145
4.6	The Modified Hotine Formula .....	145
	References .....	147
<b>5</b>	<b>Corrections in Geoid Determination</b> .....	149
5.1	Introduction .....	149
5.2	Topographic Corrections .....	150
5.2.1	The Topographic Potential and Gravity Anomaly .....	151
5.2.2	The Indirect Effect on the Geoid .....	152
5.2.3	The Combined Effect on the Geoid .....	153
5.2.4	Zero- and First-Degree Effects .....	153
5.2.5	The Topographic Bias by a Strict Formulation .....	155
5.2.6	The EGM Analytical Continuation Error (EACE) .....	159
5.2.7	The Topographic Bias in the Modified Stokes' Formula .....	164
5.2.8	Lateral Topographic Density Variations .....	166
5.3	The Downward Continuation Correction .....	167
5.3.1	The Dwc Effect on the Original Stokes' Formula .....	167
5.3.2	The Dwc Effect for the Modified Stokes' Formula .....	170
5.4	Atmospheric Corrections .....	171
5.4.1	The IAG Approach .....	171
5.4.2	The KTH Approach .....	172
5.5	Ellipsoidal Corrections .....	174
5.5.1	Components of the Ellipsoidal Correction of Stokes' Formula .....	175
5.5.2	The Ellipsoidal Correction as a Harmonic Series and a Stokes' Integral .....	176
5.6	Corrections in Quasigeoid Determination .....	179
	References .....	179