

Mathematical Analysis During the 20th Century

JEAN-PAUL PIER

Centre Universitaire de Luxembourg

OXFORD UNIVERSITY PRESS

CONTENTS

1	Introduction	1
1.1	The scope of analysis	2
<i>1.1.1</i>	The great classics on analysis	2
<i>1.1.2</i>	The changing object of analysis	4
1.2	Main streams in a turbulent activity	12
<i>1.2.1</i>	The question of subdividing mathematical analysis	12
<i>1.2.2</i>	How to organize the subject	16
2	General Topology	27
2.1	Evolution 1900–1950	27
<i>2.1.1</i>	Topological axiomatizations	27
<i>2.1.2</i>	Topological algebra	37
<i>2.1.3</i>	Filtrations	40
<i>2.1.4</i>	Dimension theory	42
<i>2.1.5</i>	Complementary inputs	42
2.2	Flashes 1950–2000	44
<i>2.2.1</i>	An accomplished subject	44
<i>2.2.2</i>	Generalized topological concepts	45
3	Integration and Measure	47
3.1	Evolution 1900–1950	47
<i>3.1.1</i>	Lebesgue integration	47
<i>3.1.2</i>	The general concept of measure	55
<i>3.1.3</i>	Paradoxical decomposition	61
<i>3.1.4</i>	Period of consolidation	61
3.2	Flashes 1950–2000	63
<i>3.2.1</i>	Standing problems	63
<i>3.2.2</i>	Abstract formulations	64
<i>3.2.3</i>	Generalized Riemann integrals	68
<i>3.2.4</i>	Outlook	69
4	Functional analysis	73
4.1	Evolution 1900–1950	73
<i>4.1.1</i>	New objectives	73
<i>4.1.2</i>	Theory of integral equations	79
<i>4.1.3</i>	Banach spaces	86
<i>4.1.4</i>	Hilbert spaces	92

	4.1.5	Von Neumann algebras	95
	4.1.6	Banach algebras	98
	4.1.7	Distributions	101
4.2	Flashes	1950–2000	106
	4.2.1	Topological vector spaces	106
	4.2.2	Extension of Weierstraß's theorem	108
	4.2.3	Fréchet spaces, Schwartz spaces, Sobolev spaces	108
	4.2.4	Banach space properties	110
	4.2.5	Hilbert space properties	113
	4.2.6	Banach algebra and C^* -algebra properties	114
	4.2.7	Approximation properties	117
	4.2.8	Nuclearity	117
	4.2.9	Von Neumann algebra properties	123
	4.2.10	Specific topics	126
5	Harmonic analysis		129
5.1	Evolution	1900–1950	129
	5.1.1	Fourier series	129
	5.1.2	Invariant measures	137
	5.1.3	Almost periodic functions	140
	5.1.4	Uniqueness of invariant measures	141
	5.1.5	Convolutions	142
	5.1.6	An evolution linked to the history of physics	145
	5.1.7	Representation theory	150
	5.1.8	Structural properties of topological groups	154
	5.1.9	Positive-definite functions	155
	5.1.10	Harmonic synthesis	157
	5.1.11	Metric locally compact Abelian groups	160
5.2	Flashes	1950–2000	163
	5.2.1	Fourier transforms	163
	5.2.2	Convolution properties	166
	5.2.3	Group representations	166
	5.2.4	Remarkable Banach algebras of functions on a locally compact group	169
	5.2.5	Specific sets	170
	5.2.6	Specific groups	171
	5.2.7	Harmonic analysis on semigroups	173
	5.2.8	Wavelets	174
	5.2.9	Generalized actions	176
6	Lie groups		179
6.1	Evolution	1900–1950	179
	6.1.1	Lie groups and Lie algebras	179
	6.1.2	Symmetric Riemannian spaces	185
	6.1.3	Hilbert's problem for Lie groups	187
	6.1.4	Representations of Lie groups	188
6.2	Flashes	1950–2000	189

6.2.1	The wide range of Lie group theory	189
6.2.2	Solution of Hilbert's problem on Lie groups	191
6.2.3	Ergodicity problems	191
6.2.4	Specific classes of Lie groups	192
6.2.5	Extensions of Lie group theory	196
7	Theory of functions and analytic geometry	199
7.1	Evolution 1900–1950	200
7.1.1	The nineteenth century continued	200
7.1.2	Potential theory	209
7.1.3	Conformal mappings	211
7.1.4	Towards a theory of several complex variables	212
7.2	Flashes 1950–2000	215
7.2.1	Accomplishments on previous topics	215
7.2.2	Hardy spaces	219
7.2.3	The dominance of the theory of several complex variables	223
7.2.4	Iteration problems	227
8	Ordinary and Partial Differential Equations	229
8.1	Evolution 1900–1950	229
8.1.1	New trends for classical problems	229
8.1.2	Fixed point properties	231
8.1.3	From the ordinary differential case to the partial differential case	232
8.2	Flashes 1950–2000	236
8.2.1	Differential equations	236
8.2.2	Partial differential equations	238
8.2.3	Tentacular subjects	247
9	Algebraic topology	255
9.1	Evolution 1900–1950	255
9.1.1	The origins of algebraic topology	255
9.1.2	Simplicial theories	261
9.1.3	Homotopy theory	267
9.1.4	Fibres and fibrations	270
9.1.5	From the ordinary differential case to the partial differential case	232
9.1.5	The breakthroughs due to Eilenberg, MacLane, and Leray	271
9.2	Flashes 1950–2000	275
9.2.1	The power of the machinery	275
9.2.2	Generalizations	278
10	Differential topology	280
10.1	Evolution 1900–1950	280
10.1.1	The beginning of the century	280
10.1.2	É. Cartan's work	283

10.1.3	Tensor products and exterior differentials	284
10.1.4	Morse theory	287
10.1.5	Whitney's work	288
10.1.6	De Rham's work	291
10.1.7	Hodge theory	293
10.1.8	The framing of the subject	297
10.2	Flashes 1950–2000	299
10.2.1	The status of differentiable manifolds	299
10.2.2	Foliations	302
10.2.3	New objectives	303
10.2.4	From Poincaré's heritage	307
10.2.5	Global analysis	310
11	Probability	314
11.1	Evolution 1900–1950	314
11.1.1	First results	314
11.1.2	Brownian motion	318
11.1.3	Ergodicity	320
11.1.4	Probabilities as measures	321
11.1.5	Stochastic integrals	324
11.2	Flashes 1950–2000	325
11.2.1	Probability theory, a part of analysis	325
11.2.2	Dynamical systems and ergodicity	326
11.2.3	Entropy	328
11.2.4	Stochastic processes	328
12	Algebraic geometry	331
12.1	Evolution 1900–1950	331
12.1.1	Algebraic geometry and number theory	331
12.1.2	The Mordell conjecture	334
12.1.3	Transcendence and prime numbers	336
12.1.4	The Riemann conjecture	338
12.2	Flashes 1950–2000	339
12.2.1	Arithmetical properties	339
12.2.2	Investigations on transcendental numbers	340
12.2.3	A central object of study	341
12.2.4	Etale cohomology	345
12.2.5	The general Riemann–Roch theorems	345
12.2.6	K -theory	346
12.2.7	Further studies	349
	References	350
	Index of Names	410
	Index of Terms	418
	Appendix: List of Symbols	427