

Contents

1	Introduction	1
2	The Exponential Transform	7
2.1	Basic Definitions	7
2.2	Moments	11
2.3	Positive Definiteness Properties	13
2.4	The Exponential Transform as a Section of a Line Bundle	16
2.5	A Riemann-Hilbert Problem	18
3	Hilbert Space Factorization	23
3.1	Definitions and Generalities	23
3.2	Restrictions and Extensions	26
3.3	Linear Operators on $\mathcal{H}(\Omega)$	27
3.4	A Functional Model for Hyponormal Operators	30
3.5	Summary in Abstract Setting	31
3.6	The Analytic Subspace $\mathcal{H}_a(\Omega)$	32
3.7	The Analytic Model	34
3.8	A Formal Comparison to Quantum Field Theory	35
3.9	Silva-Köthe-Grothendieck Duality	37
3.10	Quadrature Domains	40
3.11	Analytic Functionals	43
4	Exponential Orthogonal Polynomials	47
4.1	Orthogonal Expansions	47
4.2	Zeros of Orthogonal Polynomials	50
4.3	The Hessenberg Matrices	52
4.4	The Matrix Model of Quadrature Domains	54
5	Finite Central Truncations of Linear Operators	57
5.1	Trace Class Perturbations	57
5.2	Padé Approximation Scheme	60
5.3	Three Term Relation for the Orthogonal Polynomials	63
5.4	Disjoint Unions of Domains	66

5.5	Perturbations of Finite Truncations	68
5.6	Real Central Truncations	74
6	Mother Bodies	77
6.1	General	77
6.2	Some General Properties of Mother Bodies	81
6.3	Reduction of Inner Product to Mother Body	84
6.4	Regularity of Some Free Boundaries	86
6.5	Procedures for Finding Mother Bodies	89
7	Examples	93
7.1	The Unit Disk	93
7.2	The Annulus	94
7.3	Complements of Unbounded Quadrature Domains	95
7.3.1	The Ellipse	97
7.3.2	The Hypocycloid	99
7.4	Lemniscates	103
7.5	Polygons	105
7.5.1	Computation of Mother Body	105
7.5.2	Numerical Experiments	105
7.6	The Half-Disk, and Disk with a Sector Removed	108
7.6.1	Computation of Mother Body	108
7.6.2	Numerical Experiment	110
7.7	Domain Bounded by Two Circular Arcs	111
7.7.1	Numerical Experiment	112
7.8	External Disk	112
7.8.1	Numerical Experiment, Ellipse Plus Disk	113
7.8.2	Numerical Experiment, Pentagon Plus Disk	113
7.9	Abelian Domains	115
7.10	Disjoint Union of a Hexagon and a Hypocycloid	116
7.10.1	Numerical Experiment	116
7.11	A Square with a Disk Removed	117
7.11.1	Numerical Experiment	117
8	Comparison with Classical Function Spaces	119
8.1	Bergman Space	119
8.2	Faber Polynomials	120
A	Hyponormal Operators	125
Historical Notes	135	
Glossary	139	
References	141	
Index	147	