

Contents

Preface to the Second Edition *xi*

Preface to the First Edition *xiii*

Part I Analysis of Non Fractional Time Series 1

1	Models for Nonstationarity and Noninvertibility 3
1.1	Statistics from the One-Dimensional Random Walk 3
1.1.1	Eigenvalue Approach 4
1.1.2	Stochastic Process Approach 11
1.1.3	The Fredholm Approach 12
1.1.4	An Overview of the Three Approaches 14
1.2	A Test Statistic from a Noninvertible Moving Average Model 16
1.3	The AR Unit Root Distribution 23
1.4	Various Statistics from the Two-Dimensional Random Walk 29
1.5	Statistics from the Cointegrated Process 41
1.6	Panel Unit Root Tests 47
2	Brownian Motion and Functional Central Limit Theorems 51
2.1	The Space L_2 of Stochastic Processes 51
2.2	The Brownian Motion 55
2.3	Mean Square Integration 58
2.3.1	The Mean Square Riemann Integral 59
2.3.2	The Mean Square Riemann–Stieltjes Integral 62
2.3.3	The Mean Square Ito Integral 66
2.4	The Ito Calculus 72
2.5	Weak Convergence of Stochastic Processes 77
2.6	The Functional Central Limit Theorem 81
2.7	FCLT for Linear Processes 87
2.8	FCLT for Martingale Differences 91
2.9	Weak Convergence to the Integrated Brownian Motion 99

2.10	Weak Convergence to the Ornstein–Uhlenbeck Process	103
2.11	Weak Convergence of Vector-Valued Stochastic Processes	109
2.11.1	Space C^q	109
2.11.2	Basic FCLT for Vector Processes	110
2.11.3	FCLT for Martingale Differences	112
2.11.4	FCLT for the Vector-Valued Integrated Brownian Motion	115
2.12	Weak Convergence to the Ito Integral	118
3	The Stochastic Process Approach	127
3.1	Girsanov's Theorem: O-U Processes	127
3.2	Girsanov's Theorem: Integrated Brownian Motion	137
3.3	Girsanov's Theorem: Vector-Valued Brownian Motion	142
3.4	The Cameron–Martin Formula	145
3.5	Advantages and Disadvantages of the Present Approach	147
4	The Fredholm Approach	149
4.1	Motivating Examples	149
4.2	The Fredholm Theory: The Homogeneous Case	155
4.3	The c.f. of the Quadratic Brownian Functional	161
4.4	Various Fredholm Determinants	171
4.5	The Fredholm Theory: The Nonhomogeneous Case	190
4.5.1	Computation of the Resolvent – Case 1	192
4.5.2	Computation of the Resolvent – Case 2	199
4.6	Weak Convergence of Quadratic Forms	203
5	Numerical Integration	213
5.1	Introduction	213
5.2	Numerical Integration: The Nonnegative Case	214
5.3	Numerical Integration: The Oscillating Case	220
5.4	Numerical Integration: The General Case	228
5.5	Computation of Percent Points	236
5.6	The Saddlepoint Approximation	240
6	Estimation Problems in Nonstationary Autoregressive Models	245
6.1	Nonstationary Autoregressive Models	245
6.2	Convergence in Distribution of LSEs	250
6.2.1	Model A	251
6.2.2	Model B	253
6.2.3	Model C	255
6.2.4	Model D	257
6.3	The c.f.s for the Limiting Distributions of LSEs	260
6.3.1	The Fixed Initial Value Case	261
6.3.2	The Stationary Case	265

6.4	Tables and Figures of Limiting Distributions	267
6.5	Approximations to the Distributions of the LSEs	276
6.6	Nearly Nonstationary Seasonal AR Models	281
6.7	Continuous Record Asymptotics	289
6.8	Complex Roots on the Unit Circle	292
6.9	Autoregressive Models with Multiple Unit Roots	300
7	Estimation Problems in Noninvertible Moving Average Models	311
7.1	Noninvertible Moving Average Models	311
7.2	The Local MLE in the Stationary Case	314
7.3	The Local MLE in the Conditional Case	325
7.4	Noninvertible Seasonal Models	330
7.4.1	The Stationary Case	331
7.4.2	The Conditional Case	333
7.4.3	Continuous Record Asymptotics	335
7.5	The Pseudolocal MLE	337
7.5.1	The Stationary Case	337
7.5.2	The Conditional Case	339
7.6	Probability of the Local MLE at Unity	341
7.7	The Relationship with the State Space Model	343
8	Unit Root Tests in Autoregressive Models	349
8.1	Introduction	349
8.2	Optimal Tests	350
8.2.1	The LBI Test	352
8.2.2	The LBIU Test	353
8.3	Equivalence of the LM Test with the LBI or LBIU Test	356
8.3.1	Equivalence with the LBI Test	356
8.3.2	Equivalence with the LBIU Test	358
8.4	Various Unit Root Tests	360
8.5	Integral Expressions for the Limiting Powers	362
8.5.1	Model A	363
8.5.2	Model B	364
8.5.3	Model C	365
8.5.4	Model D	367
8.6	Limiting Power Envelopes and Point Optimal Tests	369
8.7	Computation of the Limiting Powers	372
8.8	Seasonal Unit Root Tests	382
8.9	Unit Root Tests in the Dependent Case	389
8.10	The Unit Root Testing Problem Revisited	395
8.11	Unit Root Tests with Structural Breaks	398
8.12	Stochastic Trends Versus Deterministic Trends	402
8.12.1	Case of Integrated Processes	403

13	Statistical Inference Associated with the Fractional Brownian Motion	629
13.1	Introduction	629
13.2	A Simple Continuous-Time Model Driven by the fBm	632
13.3	Quadratic Functionals of the Brownian Motion	641
13.4	Derivation of the c.f.	645
13.4.1	Stochastic Process Approach via Girsanov's Theorem	645
13.4.1.1	Case of $H = 1/2$	645
13.4.1.2	Case of $H > 1/2$	646
13.4.2	Fredholm Approach via the Fredholm Determinant	647
13.4.2.1	Case of $H = 1/2$	649
13.4.2.2	Case of $H > 1/2$	650
13.5	Martingale Approximation to the fBm	651
13.6	The Fractional Unit Root Distribution	659
13.6.1	The FD Associated with the Approximate Distribution	659
13.6.2	An Interesting Moment Property	664
13.7	The Unit Root Test Under the fBm Error	669
14	Maximum Likelihood Estimation for the Fractional Ornstein–Uhlenbeck Process	673
14.1	Introduction	673
14.2	Estimation of the Drift: Ergodic Case	677
14.2.1	Asymptotic Properties of the OLSEs	677
14.2.2	The MLE and MCE	679
14.3	Estimation of the Drift: Non-ergodic Case	687
14.3.1	Asymptotic Properties of the OLSE	687
14.3.2	The MLE	687
14.4	Estimation of the Drift: Boundary Case	692
14.4.1	Asymptotic Properties of the OLSEs	692
14.4.2	The MLE and MCE	693
14.5	Computation of Distributions and Moments of the MLE and MCE	695
14.6	The MLE-based Unit Root Test Under the fBm Error	703
14.7	Concluding Remarks	707
15	Solutions to Problems	709
	References	865
	Author Index	879
	Subject Index	883

11	Statistical Analysis of Cointegration	517
11.1	Introduction	517
11.2	Case of No Cointegration	519
11.3	Cointegration Distributions: The Independent Case	524
11.4	Cointegration Distributions: The Dependent Case	532
11.5	The Sampling Behavior of Cointegration Distributions	537
11.6	Testing for Cointegration	544
11.6.1	Tests for the Null of No Cointegration	544
11.6.2	Tests for the Null of Cointegration	547
11.7	Determination of the Cointegration Rank	552
11.8	Higher Order Cointegration	556
11.8.1	Cointegration in the $I(d)$ Case	556
11.8.2	Seasonal Cointegration	559

Part II Analysis of Fractional Time Series 567

12	ARFIMA Models and the Fractional Brownian Motion	569
12.1	Nonstationary Fractional Time Series	569
12.1.1	Case of $d = \frac{1}{2}$	570
12.1.2	Case of $d > \frac{1}{2}$	572
12.2	Testing for the Fractional Integration Order	575
12.2.1	i.i.d. Case	575
12.2.2	Dependent Case	581
12.3	Estimation for the Fractional Integration Order	584
12.3.1	i.i.d. Case	584
12.3.2	Dependent Case	586
12.4	Stationary Long-Memory Processes	591
12.5	The Fractional Brownian Motion	597
12.6	FCLT for Long-Memory Processes	603
12.7	Fractional Cointegration	608
12.7.1	Spurious Regression in the Fractional Case	609
12.7.2	Cointegrating Regression in the Fractional Case	610
12.7.3	Testing for Fractional Cointegration	614
12.8	The Wavelet Method for ARFIMA Models and the fBm	614
12.8.1	Basic Theory of the Wavelet Transform	615
12.8.2	Some Advantages of the Wavelet Transform	618
12.8.3	Some Applications of the Wavelet Analysis	625
12.8.3.1	Testing for d in ARFIMA Models	625
12.8.3.2	Testing for the Existence of Noise	626
12.8.3.3	Testing for Fractional Cointegration	627
12.8.3.4	Unit Root Tests	627

13	Statistical Inference Associated with the Fractional Brownian Motion	629
13.1	Introduction	629
13.2	A Simple Continuous-Time Model Driven by the fBm	632
13.3	Quadratic Functionals of the Brownian Motion	641
13.4	Derivation of the c.f.	645
13.4.1	Stochastic Process Approach via Girsanov's Theorem	645
13.4.1.1	Case of $H = 1/2$	645
13.4.1.2	Case of $H > 1/2$	646
13.4.2	Fredholm Approach via the Fredholm Determinant	647
13.4.2.1	Case of $H = 1/2$	649
13.4.2.2	Case of $H > 1/2$	650
13.5	Martingale Approximation to the fBm	651
13.6	The Fractional Unit Root Distribution	659
13.6.1	The FD Associated with the Approximate Distribution	659
13.6.2	An Interesting Moment Property	664
13.7	The Unit Root Test Under the fBm Error	669
14	Maximum Likelihood Estimation for the Fractional Ornstein–Uhlenbeck Process	673
14.1	Introduction	673
14.2	Estimation of the Drift: Ergodic Case	677
14.2.1	Asymptotic Properties of the OLSEs	677
14.2.2	The MLE and MCE	679
14.3	Estimation of the Drift: Non-ergodic Case	687
14.3.1	Asymptotic Properties of the OLSE	687
14.3.2	The MLE	687
14.4	Estimation of the Drift: Boundary Case	692
14.4.1	Asymptotic Properties of the OLSEs	692
14.4.2	The MLE and MCE	693
14.5	Computation of Distributions and Moments of the MLE and MCE	695
14.6	The MLE-based Unit Root Test Under the fBm Error	703
14.7	Concluding Remarks	707
15	Solutions to Problems	709
	References	865
	Author Index	879
	Subject Index	883