Contents

Preface ix

Chapter 1. Links and closed braids 1

§1.1. Links 1

§1.2. Closed braids and Alexander's theorem 3

§1.3. Braid index and writhe 10

§1.4. Stabilization, destabilization and exchange moves 11

§1.5. Braid groups 13

§1.6. Varying perspectives of closed braids 16

Exercises 18

Chapter 2. Braid foliations and Markov's theorem 21

§2.1. Two examples 22

§2.2. Braid foliation basics 27

§2.3. Obtaining braid foliations with only arcs 34

§2.4. Identifying destabilizations and stabilizations 41

§2.5. Markov's theorem for the unlink 42

§2.6. Annuli cobounded by two braids 45

§2.7. Markov's theorem 49

Exercises 50

Chapter 3. Exchange moves and Jones' conjecture 53

§3.1. Valence-two elliptic points 54

§3.2. Identifying exchange moves 56

§3.3. Reducing valence of elliptic points with changes of foliation 63
§3.4. Jones' conjecture and the generalized Jones conjecture 67
§3.5. Stabilizing to embedded annuli 68
§3.6. Euler characteristic calculations 72
§3.7. Proof of the generalized Jones conjecture 76
Exercises 80

Chapter 4. Transverse links and Bennequin's inequality 85
§4.1. Calculating the writhe and braid index 85
§4.2. The standard contact structure and transverse links 88
§4.3. The characteristic foliation and Giroux's elimination lemma 92
§4.4. Transverse Alexander theorem 95
§4.5. The self-linking number and Bennequin's inequality 97
§4.6. Tight versus overtwisted contact structures 101
§4.7. Transverse link invariants in low-dimensional topology 103
Exercises 104

Chapter 5. The transverse Markov theorem and simplicity 107
§5.1. Transverse isotopies 107
§5.2. Transverse Markov theorem 108
§5.3. Exchange reducibility implies transverse simplicity 116
§5.4. The unlink is transversely simple 120
§5.5. Torus knots are transversely simple 122
Exercises 135

Chapter 6. Botany of braids and transverse knots 137
§6.1. Infinitely many conjugacy classes 139
§6.2. Finitely many exchange equivalence classes 140
§6.3. Finitely many transverse isotopy classes 143
§6.4. Exotic botany and open questions 145
Exercises 147

Chapter 7. Flypes and transverse non-simplicity 151
§7.1. Flype templates 151
§7.2. Botany of 3-braids 154
§7.3. The clasp annulus revisited 156
§7.4. A weak MTWS for 3-braids 161
§7.5. Transverse isotopies and a transverse clasp annulus 162

§7.6. Transversely non-simple 3-braids 163
Exercises 163

Chapter 8. Arc presentations of links and braid foliations 167
§8.1. Arc presentations and grid diagrams 168
§8.2. Basic moves for arc presentations 171
§8.3. Arc presentations and braid foliations 174
§8.4. Arc presentations of the unknot and braid foliations 178
§8.5. Monotonic simplification of the unknot 181
Exercises 184

Chapter 9. Braid foliations and Legendrian links 187
§9.1. Legendrian links in the standard contact structure 187
§9.2. The Thurston-Bennequin and rotation numbers 191
§9.3. Legendrian links and grid diagrams 194
§9.4. Mirrors, Legendrian links and the grid number conjecture 198
§9.5. Steps 1 and 2 in the proof of Theorem 9.8 201
§9.6. Braided grid diagrams, braid foliations and destabilizations 204
§9.7. Step 3 in the proof of Theorem 9.8 210
Exercises 217

Chapter 10. Braid foliations and braid groups 219
§10.1. The braid group B_n 219
§10.2. The Dehornoy ordering on the braid group 221
§10.3. Braid moves and the Dehornoy ordering 223
§10.4. The Dehornoy floor and braid foliations 225
§10.5. Band generators and the Dehornoy ordering 231
§10.6. Dehornoy ordering, braid foliations and knot genus 233
Exercises 236

Chapter 11. Open book foliations 239
§11.1. Open book decompositions of 3-manifolds 239
§11.2. Open book foliations 241
§11.3. Markov's theorem in open books 242
§11.4. Change of foliation and exchange moves in open books 245
§11.5. Contact structures and open books 248
§11.6. The fractional Dehn twist coefficient 249
§11.7. Planar open book foliations and a condition on FDTC 253

§11.8. A generalized Jones conjecture for certain open books 257
Exercises 260

Chapter 12. Braid foliations and convex surface theory 263
§12.1. Convex surfaces in contact 3-manifolds 263
§12.2. Dividing sets for convex surfaces 264
§12.3. Bypasses for convex surfaces 267
§12.4. Non-thickenable solid tori 272
§12.5. Exotic botany and Legendrian invariants 277
Exercises 277

Bibliography 281

Index 287